# Package 'fdatest'

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Description Implementation of the Interval Testing Procedure for functional data in different frameworks (i.e., one or two-population frameworks, functional linear models) by means of different basis expansions (i.e., B-spline, Fourier, and phase-amplitude Fourier). The current version of the package requires functional data evaluated on a uniform grid; it automatically projects each function on a chosen functional basis; it performs the entire family of multivariate tests; and, finally, it provides the matrix of the p-values of the previous tests and the vector of the corrected p-values. The functional basis, the coupled or uncoupled scenario, and the kind of test can be chosen by the user. The package provides also a plotting function creating a graphical output of the procedure: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

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## **Description**

Implementation of the Interval Testing Procedure for functional data in different frameworks (i.e., one or two-population frameworks, functional linear models) by means of different basis expansions (i.e., B-spline, Fourier, and phase-amplitude Fourier). The current version of the package requires functional data evaluated on a uniform grid; it automatically projects each function on a chosen functional basis; it performs the entire family of multivariate tests; and, finally, it provides the matrix of the p-values of the previous tests and the vector of the corrected p-values. The functional basis, the coupled or uncoupled scenario, and the kind of test can be chosen by the user. The package provides also a plotting function creating a graphical output of the procedure: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

## **Details**

Package: fdatest Type: Package Version: 2.1

Date: 2015-02-05 License: GPL-2

## Author(s)

Alessia Pini, Simone Vantini

Maintainer: Alessia Pini <alessia.pini@polimi.it>

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

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K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

#### See Also

See also ITP1bspline, ITP1fourier, ITP2bspline, ITP2fourier, ITP2pafourier, ITP1mbspline, ITPaovbspline, and ITPimage.

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Example 1:
# Performing the ITP for one population with the Fourier basis
ITP.result <- ITP1fourier(NASAtemp$milan,maxfrequency=10,B=1000)</pre>
# Plotting the results of the ITP
plot(ITP.result)
# Plotting the p-value heatmap
ITPimage(ITP.result)
# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)</pre>
# Example 2:
# Performing the ITP for two populations with the B-spline basis
ITP.result <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=20,B=1000)</pre>
# Plotting the results of the ITP
plot(ITP.result)
# Plotting the p-values heatmap
ITPimage(ITP.result)
# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)</pre>
# Example 3:
# Fitting and testing a functional-on-scalar linear model
# Defining data and covariates
temperature <- rbind(NASAtemp$milan,NASAtemp$paris)</pre>
groups <- c(rep(0,22), rep(1,22))
# Performing the ITP
ITP.result <- ITPlmbspline(temperature ~ groups,B=1000,nknots=20,order=3)</pre>
# Summary of the ITP results
summary(ITP.result)
# Plot of the ITP results
layout(1)
```

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```
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
# All graphics on the same device
layout(matrix(1:6,nrow=3,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
```

ITP1bspline

One population Interval Testing Procedure with B-spline basis

# Description

The function implements the Interval Testing Procedure for testing the center of symmetry of a functional population evaluated on a uniform grid. Data are represented by means of the B-spline expansion and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

# Usage

```
ITP1bspline(data, mu = 0, order = 2, nknots = dim(data)[2], B = 10000)
```

# **Arguments**

data	Pointwise evaluations of the functional data set on a uniform grid. data is a matrix of dimensions c(n, J), with J evaluations on columns and n units on rows.
mu	The center of symmetry under the null hypothesis: either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.
order	Order of the B-spline basis expansion. The default is order=2.
nknots	Number of knots of the B-spline basis expansion. The default is $nknots=dim(data)[2]$ .
В	The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The defualt is B=10000.

# Value

ITP1bspline returns an object of class "ITP1".

An object of class "ITP1" is a list containing at least the following components:

basis	String vector indicating the basis used for the first phase of the algorithm. In this case equal to "B-spline".
test	String vector indicating the type of test performed. In this case equal to "1pop".
mu	Center of symmetry under the null hypothesis (as entered by the user).

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coeff Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expan-

sion. Rows are associated to units and columns to the basis index.

pval Uncorrected p-values for each basis coefficient.

pval.matrix Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The ele-

ment (i, j) of matrix pval.matrix contains the p-value of the joint NPC test

of the components (j, j+1, ..., j+(p-i)).

corrected.pval Corrected p-values for each basis coefficient.

labels Labels indicating the population membership of each data (in this case always

equal to 1).

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

heatmap.matrix Heatmap matrix of p-values (used only for plots).

#### Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

# See Also

See also ITP1fourier, ITP2bspline, ITP2fourier, ITP2pafourier, and ITP1mage.

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP for two populations with the B-spline basis
ITP.result <- ITP1bspline(NASAtemp$paris,mu=4,nknots=50,B=1000)
# Plotting the results of the ITP
plot(ITP.result,xrange=c(0,12),main='Paris temperatures')
# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(0,12))
# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)</pre>
```

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ITP1fourier	One population Interval Testing Procedure with Fourier basis

# Description

The function implements the Interval Testing Procedure for testing the center of symmetry of a functional population evaluated on a uniform grid. Data are represented by means of the Fourier expansion and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate.

# Usage

```
ITP1fourier(data, mu = 0, maxfrequency=floor(dim(data)[2]/2), B = 10000)
```

# Arguments

data	Pointwise evaluations of the functional data set on a uniform grid. data is a matrix of dimensions $c(n, J)$ , with J evaluations on columns and n units on rows.
mu	The center of symmetry under the null hypothesis: either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.
maxfrequency	The maximum frequency to be used in the Fourier basis expansion of data. The default is floor(dim(data)[2]/2), leading to an interpolating expansion.
В	The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The defualt is B=10000.

# Value

ITP1fourier returns an object of class "ITP1".

An object of class "ITP1" is a list containing at least the following components:

basis	String vector indicating the basis used for the first phase of the algorithm. In this case equal to "Fourier".
test	String vector indicating the type of test performed. In this case equal to "1pop".
mu	Center of symmetry under the null hypothesis (as entered by the user).
coeff	Matrix of dimensions $c(n,p)$ of the p coefficients of the Fourier basis expansion. Rows are associated to units and columns to the basis index: the first column is a0, the following $(p-1)/2$ columns are the ak coefficients (sine coefficients) and the last $(p-1)/2$ columns the bk coefficients (cosine coefficients).
pval	Uncorrected p-values for each frequency.
pval.matrix	Matrix of dimensions $c(p,p)$ of the p-values of the multivariate tests. The element $(i,j)$ of matrix pval.matrix contains the p-value of the joint NPC test of the components $(j,j+1,\ldots,j+(p-i))$ .

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corrected.pval Corrected p-values for each frequency.

labels Labels indicating the population membership of each data (in this case always

equal to 1).

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

heatmap.matrix Heatmap matrix of p-values (used only for plots).

## Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

#### See Also

See also ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier, and ITPimage.

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP1fourier(NASAtemp$milan,maxfrequency=20,B=1000)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')
# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(1,365))
# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)</pre>
```

ITP2bspline

Two populations Interval Testing Procedure with B-spline basis

# Description

The function implements the Interval Testing Procedure for testing the difference between two functional populations evaluated on a uniform grid. Data are represented by means of the B-spline basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

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#### Usage

```
ITP2bspline(data1, data2, mu = 0,
           order = 2, nknots = dim(data1)[2], B = 10000, paired = FALSE)
```

#### **Arguments**

data1

grid. data1 is a matrix of dimensions c(n1, J), with J evaluations on columns and n1 units on rows. data2 Pointwise evaluations of the second population's functional data set on a uniform grid. data2 is a matrix of dimensions c(n2, J), with J evaluations on columns and n2 units on rows. The difference between the first functional population and the second functional mu population under the null hypothesis. Either a constant (in this case, a constant

function is used) or a J-dimensional vector containing the evaluations on the

Pointwise evaluations of the first population's functional data set on a uniform

same grid which data are evaluated. The default is mu=0.

order Order of the B-spline basis expansion. The default is order=2.

nknots Number of knots of the B-spline basis expansion. The default is nknots=dim(data1)[2].

The number of iterations of the MC algorithm to evaluate the p-values of the R

permutation tests. The defualt is B=10000.

A logical indicating whether the test is paired. The default is FALSE. paired

#### Value

ITP2bspline returns an object of class "ITP2".

An object of class "ITP2" is a list containing at least the following components:

basis String vector indicating the basis used for the first phase of the algorithm. In this

case equal to "B-spline".

String vector indicating the type of test performed. In this case equal to "2pop". test mu

Difference between the first functional population and the second functional

population under the null hypothesis (as entered by the user).

Logical indicating whether the test is paired (as entered by the user). paired

coeff Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expan-

> sion, with n=n1+n2. Rows are associated to units and columns to the basis index. The first n1 rows report the coefficients of the first population units and the fol-

lowing n2 rows report the coefficients of the second population units

pval Uncorrected p-values for each basis coefficient.

pval.matrix Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The ele-

ment (i, j) of matrix pval.matrix contains the p-value of the joint NPC test

of the components (j, j+1, ..., j+(p-i)).

corrected.pval Corrected p-values for each basis coefficient.

labels Labels indicating the population membership of each data.

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

heatmap.matrix Heatmap matrix of p-values (used only for plots).

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## Author(s)

Alessia Pini, Simone Vantini

## References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

#### See Also

For tests of comparison between two populations, see ITP2fourier, ITP2pafourier. For different types of ITP-based tests, see ITP1bspline, ITP1fourier, ITP1mbspline, ITPaovbspline and ITPimage.

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=50,B=1000)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')
# Plotting the p-values heatmap
ITPimage(ITP.result,abscissa.range=c(0,12))
# Selecting the significant components at 5% level
which(ITP.result$corrected.pval < 0.05)</pre>
```

ITP2fourier

Two populations Interval Testing Procedure with Fourier basis

# **Description**

The function implements the Interval Testing Procedure for testing the difference between two functional populations evaluated on a uniform grid. Data are represented by means of the Fourier basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate.

## Usage

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#### **Arguments**

data1 Pointwise evaluations of the first population's functional data set on a uniform

grid. data1 is a matrix of dimensions c(n1, J), with J evaluations on columns

and n1 units on rows.

data2 Pointwise evaluations of the second population's functional data set on a uni-

form grid. data2 is a matrix of dimensions c(n2, J), with J evaluations on

columns and n2 units on rows.

mu The difference between the first functional population and the second functional

population under the null hypothesis. Either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the

same grid which data are evaluated. The default is mu=0.

maxfrequency The maximum frequency to be used in the Fourier basis expansion of data. The

default is floor(dim(data1)[2]/2), leading to an interpolating expansion.

B The number of iterations of the MC algorithm to evaluate the p-values of the

permutation tests. The defualt is B=10000.

paired A logical indicating whether the test is paired. The default is FALSE.

#### Value

ITP2fourier returns an object of class "ITP2".

An object of class "ITP2" is a list containing at least the following components:

basis String vector indicating the basis used for the first phase of the algorithm. In this

case equal to "Fourier".

test String vector indicating the type of test performed. in this case equal to "2pop".

mu Difference between the first functional population and the second functional

population under the null hypothesis (as entered by the user).

paired Logical indicating whether the test is paired (as entered by the user).

coeff Matrix of dimensions c(n,p) of the p coefficients of the Fourier basis expan-

sion. Rows are associated to units and columns to the basis index: the first n1 rows report the coefficients of the first population units and the following n2 rows report the coefficients of the second population units; the first column is a0, the following (p-1)/2 columns are the ak coefficients (sine coefficients) and

the last (p-1)/2 columns the bk coefficients (cosine coefficients).

pval Uncorrected p-values for each frequency.

pval.matrix Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The ele-

ment (i, j) of matrix pval.matrix contains the p-value of the joint NPC test

of the frequencies (j, j+1, ..., j+(p-i)).

corrected.pval Corrected p-values for each frequency.

labels Labels indicating the population membership of each data.

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

heatmap.matrix Heatmap matrix of p-values (used only for plots).

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## Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

#### See Also

See also ITP2pafourier, ITP2bspline, ITP1fourier, ITP1bspline, and ITPimage.

## **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP2fourier(NASAtemp$milan,NASAtemp$paris,maxfrequency=20,B=1000,paired=TRUE)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')
# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(1,365))
# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)</pre>
```

ITP2pafourier

Two populations Interval Testing Procedure with Fourier basis (phase-amplitude decomposition)

## **Description**

The function implements the Interval Testing Procedure for testing the difference between two functional populations evaluated on a uniform grid. Data are represented by means of the Fourier basis expansion with the phase-amplitude decomposition and the significance of the amplitude and phase of each frequency is tested with an interval-wise control of the Family Wise Error Rate.

## Usage

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#### **Arguments**

data1 Pointwise evaluations of the first population's functional data set on a uniform

grid. data1 is a matrix of dimensions c(n1, J), with J evaluations on columns

and n1 units on rows.

data2 Pointwise evaluations of the second population's functional data set on a uni-

form grid. data2 is a matrix of dimensions c(n2, J), with J evaluations on

columns and n2 units on rows.

maxfrequency The maximum frequency to be used in the Fourier basis expansion of data. The

default is floor(dim(data1)[2]/2), leading to an interpolating expansion.

B The number of iterations of the MC algorithm to evaluate the p-values of the

permutation tests. The defualt is B=10000.

paired A logical indicating whether the test is paired. The default is FALSE.

#### Value

ITP2pafourier returns an object of class "ITP2".

An object of class "ITP2" is a list containing at least the following components:

basis String vector indicating the basis used for the first phase of the algorithm. Equal

to "paFourier".

test String vector indicating the type of test performed. Equal to "2pop".

Logical indicating whether the test is paired (as entered by the user).

coeff\_phase Matrix of dimensions c(n,p) of the p phases of the Fourier basis expansion.

Rows are associated to units and columns to frequencies: the first n1 rows report the coefficients of the first population units and the following n2 rows report the

coefficients of the second population units.

coeff\_amplitude

Matrix of dimensions c(n,p) of the p amplitudes of the Fourier basis expansion. Rows are associated to units and columns to frequencies: the first n1 rows report the coefficients of the first population units and the following n2 rows report the

coefficients of the second population units.

pval\_phase Uncorrected p-values of the phase tests for each frequency.

pval\_amplitude Uncorrected p-values of the amplitude tests for each frequency.

pval.matrix\_phase

Matrix of dimensions c(p,p) of the p-values of the multivariate tests on phase. The element (i,j) of matrix pval.matrix\_phase contains the p-value of the

joint NPC test of the frequencies (j, j+1, ..., j+(p-i)).

pval.matrix\_amplitude

Matrix of dimensions c(p,p) of the p-values of the multivariate tests on amplitude. The element (i,j) of matrix pval.matrix\_amplitude contains the p-value of the joint NPC test of the frequencies  $(j,j+1,\ldots,j+(p-i))$ .

corrected.pval\_phase

Corrected p-values of the phase tests for each frequency.

corrected.pval\_amplitude

Corrected p-values of the amplitude tests for each frequency.

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labels Labels indicating the population membership of each data.

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

heatmap.matrix\_phase

Heatmap matrix of p-values for phase (used only for plots).

heatmap.matrix\_amplitude

Heatmap matrix of p-values for amplitude (used only for plots).

## Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

## See Also

See also ITP2fourier, ITP2bspline, ITP1fourier, ITP1bspline, and ITPimage.

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP2pafourier(NASAtemp$milan,NASAtemp$paris,maxfrequency=20,B=1000,paired=TRUE)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')
# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(1,365))
# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)</pre>
```

ITPaovbspline

Interval Testing Procedure for testing Functional analysis of variance with B-spline basis

# **Description**

ITPaovbspline is used to fit and test functional analysis of variance. The function implements the Interval Testing Procedure for testing for significant differences between several functional population evaluated on a uniform grid. Data are represented by means of the B-spline basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

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#### Usage

## **Arguments**

formula An object of class "formula" (or one that can be coerced to that class): a sym-

bolic description of the model to be fitted.

order Order of the B-spline basis expansion. The default is order=2.

nknots Number of knots of the B-spline basis expansion.

The default is dim(model.response(model.frame(formula)))[2].

B The number of iterations of the MC algorithm to evaluate the p-values of the

permutation tests. The defualt is B=10000.

method Permutation method used to calculate the p-value of permutation tests. Choose

"residuals" for the permutations of residuals under the reduced model, according to the Freedman and Lane scheme, and "responses" for the permutation of

the responses, according to the Manly scheme.

#### Value

ITPaovbspline returns an object of class "ITPaov".

The function summary is used to obtain and print a summary of the results.

An object of class "ITP1m" is a list containing at least the following components:

call The matched call.

design.matrix The design matrix of the functional-on-scalar linear model.

basis String vector indicating the basis used for the first phase of the algorithm. In this

case equal to "B-spline".

coeff Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expan-

sion. Rows are associated to units and columns to the basis index.

coeff.regr Matrix of dimensions c(L+1,p) of the p coefficients of the B-spline basis ex-

pansion of the intercept (first row) and the L effects of the covariates specified

in formula. Columns are associated to the basis index.

pval.F Uncorrected p-values of the functional F-test for each basis coefficient.

pval.matrix.F Matrix of dimensions c(p,p) of the p-values of the multivariate F-tests. The

element (i,j) of matrix pval.matrix contains the p-value of the joint NPC

test of the components (j, j+1, ..., j+(p-i)).

corrected.pval.F

Corrected p-values of the functional F-test for each basis coefficient.

pval.factors Uncorrected p-values of the functional F-tests on each factor of the analysis of

variance, separately (rows) and each basis coefficient (columns).

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pval.matrix.factors

Array of dimensions c(L+1,p,p) of the p-values of the multivariate F-tests on factors. The element (1,i,j) of array pval.matrix contains the p-value of the joint NPC test on factor 1 of the components  $(j,j+1,\ldots,j+(p-i))$ .

corrected.pval.factors

Corrected p-values of the functional F-tests on each factor of the analysis of variance (rows) and each basis coefficient (columns).

Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.

coeff.regr.eval

Evaluation on a fine uniform grid of the functional regression coefficients.

fitted.eval Evaluation on a fine uniform grid of the fitted values of the functional regression.

residuals.eval Evaluation on a fine uniform grid of the residuals of the functional regression.

R2. eval Evaluation on a fine uniform grid of the functional R-squared of the regression.

heatmap.matrix.F

Heatmap matrix of p-values of functional F-test (used only for plots).

heatmap.matrix.factors

Heatmap matrix of p-values of functional F-tests on each factor of the analysis of variance (used only for plots).

## Author(s)

Alessia Pini, Simone Vantini

#### References

D. Freedman and D. Lane (1983). A Nonstochastic Interpretation of Reported Significance Levels. *Journal of Business & Economic Statistics* 1.4, 292-298.

B. F. J. Manly (2006). Randomization, *Bootstrap and Monte Carlo Methods in Biology*. Vol. 70. CRC Press.

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

## See Also

See summary. ITPaov for summaries and plot. ITPaov for plotting the results.

See also ITPlmbspline to fit and test a functional-on-scalar linear model applying the ITP, and ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier for one-population and two-population tests.

ITPimage

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups,B=1000,nknots=20,order=3)

# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
layout(1)
plot(ITP.result)

# All graphics on the same device
layout(matrix(1:4,nrow=2,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))</pre>
```

**ITPimage** 

Plot of the Interval Testing Procedure results

#### **Description**

Plotting function creating a graphical output of the ITP: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

# Usage

```
ITPimage(ITP.result, alpha = 0.05, abscissa.range = c(0, 1), nlevel = 20)
```

## **Arguments**

ITP.result Results of the ITP, as created by ITP1bspline, ITP1fourier, ITP2bspline, ITP2fourier, and ITP2pafourier.

alpha Level of the hypothesis test. The default is alpha=0.05.

abscissa.range Range of the plot abscissa. The default is c(0,1).

Number of desired color levels for the p-value heatmap. The default is nlevel=20.

#### Value

No value returned. The function produces a graphical output of the ITP results: the p-value heatmap, a plot of the corrected p-values and the plot of the functional data. The basis components selected as significant by the test at level alpha are highlighted in the plot of the corrected p-values by a gray area.

ITPImbspline 17

#### Author(s)

Alessia pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

#### See Also

See plot. ITP1, plot. ITP2, plot. ITP1m, and plot. ITPaov for the plot method applied to the ITP results of one- and two-population tests, linear models, and ANOVA, respectively.

See also ITP1bspline, ITP1fourier, ITP2bspline, ITP2fourier, and ITP2pafourier for applying the ITP.

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for two populations with the B-spline basis
ITP.result <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=20,B=1000)

# Plotting the results of the ITP
ITPimage(ITP.result,abscissa.range=c(0,12))

# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)</pre>
```

ITPlmbspline

Interval Testing Procedure for testing Functional-on-Scalar Linear Models with B-spline basis

# Description

ITPlmbspline is used to fit and test functional linear models. It can be used to carry out regression, and analysis of variance. The function implements the Interval Testing Procedure for testing the significance of the effects of scalar covariates on a functional population evaluated on a uniform grid. Data are represented by means of the B-spline basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

# Usage

#### **Arguments**

formula An object of class "formula" (or one that can be coerced to that class): a sym-

bolic description of the model to be fitted.

order Order of the B-spline basis expansion. The default is order=2.

nknots Number of knots of the B-spline basis expansion.

The default is dim(model.response(model.frame(formula)))[2].

B The number of iterations of the MC algorithm to evaluate the p-values of the

permutation tests. The defualt is B=10000.

method Permutation method used to calculate the p-value of permutation tests. Choose

"residuals" for the permutations of residuals under the reduced model, according to the Freedman and Lane scheme, and "responses" for the permutation of

the responses, according to the Manly scheme.

#### Value

ITPlmbspline returns an object of class "ITPlm".

The function summary is used to obtain and print a summary of the results.

An object of class "ITPlm" is a list containing at least the following components:

call The matched call.

design.matrix The design matrix of the functional-on-scalar linear model.

basis String vector indicating the basis used for the first phase of the algorithm. In this

case equal to "B-spline".

coeff Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expan-

sion. Rows are associated to units and columns to the basis index.

coeff.regr Matrix of dimensions c(L+1,p) of the p coefficients of the B-spline basis ex-

pansion of the intercept (first row) and the L effects of the covariates specified

in formula. Columns are associated to the basis index.

pval.F Uncorrected p-values of the functional F-test for each basis coefficient.

pval.matrix.F Matrix of dimensions c(p,p) of the p-values of the multivariate F-tests. The

element (i, j) of matrix pval.matrix contains the p-value of the joint NPC

test of the components (j, j+1, ..., j+(p-i)).

corrected.pval.F

Corrected p-values of the functional F-test for each basis coefficient.

pval.t Uncorrected p-values of the functional t-tests for each partial regression coeffi-

cient including the intercept (rows) and each basis coefficient (columns).

pval.matrix.t Array of dimensions c(L+1,p,p) of the p-values of the multivariate t-tests. The

element (l,i,j) of array pval.matrix contains the p-value of the joint NPC

test on covariate 1 of the components (j, j+1, ..., j+(p-i)).

corrected.pval.t

Corrected p-values of the functional t-tests for each partial regression coefficient

including the intercept (rows) and each basis coefficient (columns).

data.eval Evaluation on a fine uniform grid of the functional data obtained through the

basis expansion.

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```
coeff.regr.eval
```

Evaluation on a fine uniform grid of the functional regression coefficients.

fitted.eval Evaluation on a fine uniform grid of the fitted values of the functional regression.

residuals.eval Evaluation on a fine uniform grid of the residuals of the functional regression.

R2.eval Evaluation on a fine uniform grid of the functional R-squared of the regression.

heatmap.matrix.F

Heatmap matrix of p-values of functional F-test (used only for plots).

heatmap.matrix.t

Heatmap matrix of p-values of functional t-tests (used only for plots).

## Author(s)

Alessia Pini, Simone Vantini

#### References

D. Freedman and D. Lane (1983). A Nonstochastic Interpretation of Reported Significance Levels. *Journal of Business & Economic Statistics* 1.4, 292-298.

B. F. J. Manly (2006). Randomization, *Bootstrap and Monte Carlo Methods in Biology*. Vol. 70. CRC Press.

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

# See Also

See summary. ITPlm for summaries and plot. ITPlm for plotting the results.

See also ITPaovbspline to fit and test a functional analysis of variance applying the ITP, and ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier for one-population and two-population tests.

```
# Importing the NASA temperatures data set
data(NASAtemp)

# Defining the covariates
temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlmbspline(temperature ~ groups,B=1000,nknots=20)
# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results</pre>
```

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```
layout(1)
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
# All graphics on the same device
layout(matrix(1:6,nrow=3,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
```

**NASAtemp** 

NASA daily temperatures data set

#### **Description**

It contains the daily mean temperatures registered from July 1983 to June 2005 and stored in the NASA database Earth Surface Meteorology for Solar Energy of two different geographical locations: the region (45-46 North, 9-10 East), including the city of Milan (Italy), and the region (48-49 North, 2-3 East), including the city of Paris (France).

#### Usage

```
data(NASAtemp)
```

#### **Format**

List of 2 elements:

- milan Matrix of dimensions c(22, 365) containing the daily mean temperatures of the region (45-46 North, 9-10 East), including the city of Milan (Italy) registered from July 1983 to June 2005 (22 years).
- paris Matrix of dimensions c(22, 365) containing the daily mean temperatures of the region (48-49 North, 2-3 East), including the city of Paris (France) registered from July 1983 to June 2005 (22 years).

#### Source

These data were obtained from the NASA Langley Research Center Atmospheric Science Data Center Surface meteorological and Solar Energy (SSE) web portal supported by the NASA LaRC POWER Project. Data are freely available at: NASA Surface Meteorology and Solar Energy, A Renewable Energy Resource web site (release 6.0): http://eosweb.larc.nasa.gov

```
data(NASAtemp)
## Not run:
matplot(t(NASAtemp$milan),type='l')
matplot(t(NASAtemp$paris),type='l')
```

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```
## End(Not run)
```

plot.ITP1

Plotting ITP results for one-population tests

## **Description**

plot method for class "ITP1". Plotting function creating a graphical output of the ITP for the test of the mean of one population: functional data and ITP-adjusted p-values are plotted.

# Usage

```
## S3 method for class 'ITP1'
plot(x, xrange = c(0, 1), alpha1 = 0.05, alpha2 = 0.01,
    ylab = "Functional Data", main = NULL, lwd = 1, col = 1,
    pch = 16, ylim = range(object$data.eval), ...)
```

#### **Arguments**

х	The object to be plotted. An object of class "ITP1", that is, a result of an ITP for comparison between two populations. Usually a call to ITP1bspline or ITP1fourier.
xrange	Range of the x axis.
alpha1	First level of significance used to select and display significant differences. Default is alpha1 = 0.05.
alpha2	Second level of significance used to select and display significant differences. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
ylab	Label of y axis of the plot of functional data. Default is "Functional Data".
main	An overall title for the plots (it will be pasted to "Functional Data" for the first plot and "adjusted p-values" for the second plot).
lwd	Line width for the plot of functional data.
col	Color used to plot the functional data.
pch	Point character for the plot of adjusted p-values.
ylim	Range of the y axis.
• • •	Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

# Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data and the one of the adjusted p-values. The basis components selected as significant by the test at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data (in case the test is based on a local basis, such as B-splines) by gray areas (light and dark gray, respectively).

22 plot.ITP2

#### Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

#### See Also

ITPimage for the plot of p-values heatmaps.

See also ITP1bspline and ITP1fourier to perform the ITP to test for the mean of a functional populations. See plot.ITP2 and plot.ITP1m for the plot method applied to the ITP results of two-population tests and linear models, respectively.

# **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for one population with the B-spline basis
ITP.result.bspline <- ITP1bspline(NASAtemp$paris,mu=4,nknots=50,B=1000)
# Plotting the results of the ITP
plot(ITP.result.bspline,xlab='Day',xrange=c(0,365),main='NASA data')
# Selecting the significant components for the radius at 5% level
which(ITP.result.bspline$corrected.pval < 0.05)</pre>
```

plot.ITP2

Plotting ITP results for two-population tests

#### **Description**

plot method for class "ITP2". Plotting function creating a graphical output of the ITP for the test of comparison between two populations: functional data and ITP-adjusted p-values are plotted.

#### Usage

```
## S3 method for class 'ITP2'
plot(x, xrange = c(0, 1), alpha1 = 0.05, alpha2 = 0.01,
    ylab = "Functional Data", main = NULL, lwd = 1,
    col = c(1, 2), pch = 16, ylim = range(object$data.eval), ...)
```

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Arguments
-----------

Х	The object to be plotted. An object of class "ITP2", that is, a result of an ITP for comparison between two populations. Usually a call to ITP2bspline, ITP2fourier or ITP2pafourier.
xrange	Range of the x axis.
alpha1	First level of significance used to select and display significant differences. Default is alpha1 = 0.05.
alpha2	Second level of significance used to select and display significant differences. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
ylab	Label of y axis of the plot of functional data. Default is "Functional Data".
main	An overall title for the plots (it will be pasted to "Functional Data" for the first plot and "adjusted p-values" for the second plot).
lwd	Line width for the plot of functional data.
col	Color used to plot the functional data.
pch	Point character for the plot of adjusted p-values.
ylim	Range of the y axis.
•••	Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

# Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data and the one of the adjusted p-values. The basis components selected as significant by the test at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data (in case the test is based on a local basis, such as B-splines) by gray areas (light and dark gray, respectively). In the case of a Fourier basis with amplitude and phase decomposition, two plots of adjusted p-values are done, one for phase and one for amplitude.

## Author(s)

Alessia Pini, Simone Vantini

# References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

# See Also

ITPimage for the plot of p-values heatmaps.

See also ITP2bspline, ITP2fourier, ITP2pafourier to perform the ITP to test for differences between two populations. See plot.ITP1 and plot.ITP1m for the plot method applied to the ITP results of one-population tests and a linear models, respectively.

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## **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for two populations with the B-spline basis
ITP.result.bspline <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=30,B=1000)
# Plotting the results of the ITP
plot(ITP.result.bspline,xlab='Day',xrange=c(1,365),main='NASA data')

# Selecting the significant components for the radius at 5% level
which(ITP.result.bspline$corrected.pval < 0.05)</pre>
```

plot.ITPaov

Plotting ITP results for functional analysis of variance testing

# **Description**

plot method for class "ITPaov". Plotting function creating a graphical output of the ITP for the test on a functional analysis of variance: functional data, and ITP-adjusted p-values of the F-tests on the whole model and on each factor are plotted.

## Usage

# Arguments

х	The object to be plotted. An object of class "ITPaov", usually, a result of a call to ITPaovbspline.
xrange	Range of the x axis.
alpha1	First level of significance used to select and display significant effects. Default is alpha1 = 0.05.
alpha2	Second level of significance used to select and display significant effects. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
plot.adjpval	A logical indicating wether the plots of adjusted p-values have to be done. Default is plot.adjpval = FALSE.
col	Colors for the plot of functional data. Default is col = 1.
ylim	Range of the y axis. Default is ylim = range(x\$data.eval).
ylab	Label of y axis of the plot of functional data. Default is "Functional Data".

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main	An overall title for the plots (it will be pasted to "Functional Data and F-test" for the first plot and "factor" for the other plots).	
lwd	Line width for the plot of functional data. Default is lwd=16.	
pch	Point character for the plot of adjusted p-values. Default is pch=16.	
	Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).	

#### Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data, functional regression coefficients, and ITP-adjusted p-values of the F-tests on the whole model and on each factor. The basis components selected as significant by the tests at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data by gray areas (light and dark gray, respectively). The first plot reports the gray areas corresponding to a significant F-test on the whole model. The remaining plots report the gray areas corresponding to significant F-tests on each factor (with colors corresponding to the levels of the factor).

## Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

#### See Also

See also ITPaovbspline to fit and test a functional analysis of variance applying the ITP, and summary.ITPaov for summaries. See plot.ITPlm, plot.ITP1, and plot.ITP2 for the plot method applied to the ITP results of functional-on-scalar linear models, one-population and two-population, respectively.

```
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups,B=1000,nknots=20,order=3)

# Summary of the ITP results
summary(ITP.result)</pre>
```

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```
# Plot of the ITP results
layout(1)
plot(ITP.result)

# All graphics on the same device
layout(matrix(1:4,nrow=2,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
```

plot.ITPlm

Plotting ITP results for functional-on-scalar linear model testing

## **Description**

plot method for class "ITPlm". Plotting function creating a graphical output of the ITP for the test on a functional-on-scalar linear model: functional data, functional coefficients and ITP-adjusted p-values for the F-test and t-tests are plotted.

## Usage

```
## S3 method for class 'ITPlm'
plot(x, xrange = c(0, 1), alpha1 = 0.05, alpha2 = 0.01,
    plot.adjpval = FALSE, col = c(1, rainbow(dim(x$corrected.pval.t)[1])),
    ylim = range(x$data.eval), ylab = "Functional Data",
    main = NULL, lwd = 1, pch = 16, ...)
```

# **Arguments**

X	The object to be plotted. An object of class "ITPlm", usually, a result of a call to ITPlmbspline.
xrange	Range of the x axis.
alpha1	First level of significance used to select and display significant effects. Default is alpha1 = 0.05.
alpha2	Second level of significance used to select and display significant effects. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
plot.adjpval	A logical indicating wether the plots of adjusted p-values have to be done. Default is plot.adjpval = FALSE.
col	Vector of colors for the plot of functional data (first element), and functional coefficients (following elements).
	Default is col = $c(1, rainbow(dim(x\$corrected.pval.t)[1]))$ .
ylim	Range of the y axis. Default is ylim = range(x\$data.eval).
ylab	Label of y axis of the plot of functional data. Default is "Functional Data".
main	An overall title for the plots (it will be pasted to "Functional Data and F-test" for the first plot and "t-test" for the other plots).

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lwd	Line width for the plot of functional data. Default is 1wd=16.
pch	Point character for the plot of adjusted p-values. Default is pch=16.
• • •	Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

## Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data, functional regression coefficients, and ITP-adjusted p-values for the F-test and t-tests. The basis components selected as significant by the tests at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data by gray areas (light and dark gray, respectively). The plot of functional data reports the gray areas corresponding to a significant F-test. The plots of functional regression coefficients report the gray areas corresponding to significant t-tests for the corresponding covariate.

# Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

## See Also

See also ITPlmbspline to fit and test a functional-on-scalar linear model applying the ITP, and summary.ITPlm for summaries. See plot.ITPaov, plot.ITP1, and plot.ITP2 for the plot method applied to the ITP results of functional analysis of variance, one-population and two-population, respectively.

```
# Importing the NASA temperatures data set
data(NASAtemp)

data <- rbind(NASAtemp$milan,NASAtemp$paris)
lab <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlmbspline(data ~ lab,B=1000,nknots=20,order=3)
# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
layout(1)
plot(ITP.result,main='NASA data',xlab='Day',xrange=c(1,365))</pre>
```

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```
# Plots of the adjusted p-values
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
# To have all plots in one device
layout(matrix(1:6,nrow=3,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE,xlab='Day',xrange=c(1,365))
```

summary.ITPaov

Summarizing Functional Analysis of Variance Fits

# **Description**

summary method for class "ITPaov".

# Usage

```
## S3 method for class 'ITPaov'
summary(object, ...)
```

# **Arguments**

object An object of class "ITPaov", usually, a result of a call to ITPaovbspline.

... Further arguments passed to or from other methods.

# Value

The function summary. ITPaov computes and returns a list of summary statistics of the fitted functional analysis of variance given in object, using the component "call" from its arguments, plus:

factors A L x 1 matrix with columns for the factors of ANOVA, and corresponding

(two-sided) ITP-adjusted minimum p-values of the corresponding tests of significance (i.e., the minimum p-value over all p basis components used to describe

functional data).

R2 Range of the functional R-squared.

ftest ITP-adjusted minimum p-value of functional F-test.

#### Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

summary.ITPlm 29

## See Also

See ITPaovbspline for fitting and testing the functional ANOVA and plot.ITPaov for plots. See also ITPlmbspline, ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier.

#### **Examples**

```
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups,B=1000,nknots=20,order=3)

# Summary of the ITP results
summary(ITP.result)</pre>
```

summary.ITPlm

Summarizing Functional-on-Scalar Linear Model Fits

# **Description**

summary method for class "ITPlm".

# Usage

```
## S3 method for class 'ITPlm'
summary(object, ...)
```

## **Arguments**

object An object of class "ITPlm", usually, a result of a call to ITPlmbspline.

... Further arguments passed to or from other methods.

# Value

The function summary. ITPlm computes and returns a list of summary statistics of the fitted functional-on-scalar linear model given in object, using the component "call" from its arguments, plus:

ttest A L+1 x 1 matrix with columns for the functional regression coefficients, and

corresponding (two-sided) ITP-adjusted minimum p-values of t-tests (i.e., the minimum p-value over all p basis components used to describe functional data).

R2 Range of the functional R-squared.

ftest ITP-adjusted minimum p-value of functional F-test.

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## Author(s)

Alessia Pini, Simone Vantini

#### References

A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Controlling the Family Wise Error Rate on Intervals. *MOX-report 13/2013*, Politecnico di Milano.

K. Abramowicz, S. De Luna, C. Häger, A. Pini, L. Schelin, and S. Vantini (2015). Distribution-Free Interval-Wise Inference for Functional-on-Scalar Linear Models. *MOX-report 3/2015*, Politecnico di Milano.

## See Also

See ITPlmbspline for fitting and testing the functional linear model and plot.ITPlm for plots. See also ITPaovbspline, ITP1bspline, ITP2bspline, ITP2bspline, ITP2pafourier.

```
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlmbspline(temperature ~ groups, B=1000,nknots=20)

# Summary of the ITP results
summary(ITP.result)</pre>
```

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