Package ‘fracdiff’

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Title Fractionally Differenced ARIMA aka ARFIMA(P,d,q) Models

Description Maximum likelihood estimation of the parameters of a fractionally differenced ARIMA(p,d,q) model (Haslett and Raftery, Appl.Statistics, 1989); including inference and basic methods. Some alternative algorithms to estimate ``H''.

Imports stats

Suggests longmemo, forecast, urca

License GPL (>= 2)

URL https://github.com/mmaechler/fracdiff

BugReports https://github.com/mmaechler/fracdiff/issues

Encoding UTF-8

NeedsCompilation yes

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R topics documented:

`confint.fracdiff` *Confidence Intervals for Fracdiff Model Parameters*

Description

Computes (Wald) confidence intervals for one or more parameters in a fitted fracdiff model, see `fracdiff`.

Usage

```
## S3 method for class 'fracdiff'
confint(object, parm, level = 0.95, ...)
```

Arguments

`object`	an object of class `fracdiff`, typically result of `fracdiff(..)`.
`parm`	a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
`level`	the confidence level required.
`...`	additional argument(s) for methods.

Value

A matrix (or vector) with columns giving lower and upper confidence limits for each parameter. These will be labelled as (1-level)/2 and 1 - (1-level)/2 in % (by default 2.5% and 97.5%).

Warning

As these confidence intervals use the standard errors returned by `fracdiff()` (which are based on finite difference approximations to the Hessian) they may end up being much too narrow, see the example in `fracdiff.var`.

Author(s)

Spencer Graves posted the initial version to R-help.

See Also

the generic `confint`; `fracdiff` model fitting, notably `fracdiff.var()` for re-estimating the variance-covariance matrix on which `confint()` builds entirely.

Examples

```
set.seed(101)
ts2 <- fracdiff.sim(5000, ar = .2, ma = -.4, d = .3)
mFD <- fracdiff( ts2$series, nar = length(ts2$ar), nma = length(ts2$ma))
coef(mFD)
confint(mFD)
```

`diffseries` *Fractionally Differenciate Data*

Description

Differenciates the time series data using the approximated binomial expression of the long-memory filter and an estimate of the memory parameter in the ARFIMA(p,d,q) model.

Usage

```
diffseries(x, d)
```

Arguments

`x`	numeric vector or univariate time series.
`d`	number specifiying the fractional difference order.

Details

Since 2018, we are using (an important correction of) the fast algorithm based on the discrete Fourier transform (`fft`) by Jensen and Nielsen which is significantly faster for large `n = length(x)`.

Value

the fractionally differenced series `x`.

Author(s)

Valderio A. Reisen `<valderio@cce.ufes.br>` and Artur J. Lemonte (first slow version), now hidden as `diffseries.0()`.

Current version: Jensen and Nielsen (2014); tweaks by Martin Maechler, 2018.

References

See those in fdSperio; additionally

Reisen, V. A. and Lopes, S. (1999) Some simulations and applications of forecasting long-memory time series models; *Journal of Statistical Planning and Inference* **80**, 269–287.

Reisen, V. A. Cribari-Neto, F. and Jensen, M.J. (2003) Long Memory Inflationary Dynamics. The case of Brazil. *Studies in Nonlinear Dynamics and Econometrics* **7**(3), 1–16.

Jensen, Andreas Noack and Nielsen, Morten Ørregaard (2014) A Fast Fractional Difference Algorithm. *Journal of Time Series Analysis* **35**(5), 428–436; doi: 10.1111/jtsa.12074.

See Also

fracdiff.sim

Examples

```
memory.long <- fracdiff.sim(80, d = 0.3)
str(mGPH <- fdGPH(memory.long$series))
r <- diffseries(memory.long$series, d = mGPH$d)
#acf(r) # shouldn't show structure - ideally
```

---

fdGPH *Geweke and Porter-Hudak Estimator for ARFIMA(p,d,q)*

Description

Estimate the fractional (or “memory”) parameter d in the ARFIMA(p,d,q) model by the method of Geweke and Porter-Hudak (GPH). The GPH estimator is based on the regression equation using the periodogram function as an estimate of the spectral density.

Usage

```
fdGPH(x, bandw.exp = 0.5)
```

Arguments

`x`	univariate time series
`bandw.exp`	the bandwidth used in the regression equation

Details

The function also provides the asymptotic standard deviation and the standard error deviation of the fractional estimator.

The bandwidth is `bw = trunc(n ^ bandw.exp)`, where $0 <$ bandw.exp < 1 and n is the sample size. Default `bandw.exp = 0.5`.

Value

d	GPH estimate
sd.as	asymptotic standard deviation
sd.reg	standard error deviation

Author(s)

Valderio A. Reisen and Artur J. Lemonte

References

see those in `fdSperio`.

See Also

`fdSperio`, `fracdiff`

Examples

```
memory.long <- fracdiff.sim(1500, d = 0.3)
fdGPH(memory.long$series)
```

`fdSperio` *Sperio Estimate for 'd' in ARFIMA(p,d,q)*

Description

This function makes use Reisen (1994) estimator to estimate the memory parameter d in the ARFIMA(p,d,q) model. It is based on the regression equation using the smoothed periodogram function as an estimate of the spectral density.

Usage

```
fdSperio(x, bandw.exp = 0.5, beta = 0.9)
```

Arguments

x	univariate time series data.
bandw.exp	numeric: exponent of the bandwidth used in the regression equation.
beta	numeric: exponent of the bandwidth used in the lag Parzen window.

Details

The function also provides the asymptotic standard deviation and the standard error deviation of the fractional estimator.

The bandwidths are bw = `trunc(n ^ bandw.exp)`, where $0 < \text{bandw.exp} < 1$ and n is the sample size. Default `bandw.exp= 0.5`;
and bw2 = `trunc(n ^ beta)`, where $0 < \text{beta} < 1$ and n is the sample size. Default `beta = 0.9`.

Value

a list with components

`d`	Sperio estimate
`sd.as`	asymptotic standard deviation
`sd.reg`	standard error deviation

Author(s)

Valderio A. Reisen <valderio@cce.ufes.br> and Artur J. Lemonte

References

Geweke, J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models. *Journal of Time Series Analysis* **4**(4), 221–238.

Reisen, V. A. (1994) Estimation of the fractional difference parameter in the ARFIMA(p,d,q) model using the smoothed periodogram. *Journal Time Series Analysis*, **15**(1), 335–350.

Reisen, V. A., B. Abraham, and E. M. M. Toscano (2001) Parametric and semiparametric estimations of stationary univariate ARFIMA model. *Brazilian Journal of Probability and Statistics* **14**, 185–206.

See Also

`fdGPH`, `fracdiff`

Examples

```
memory.long <- fracdiff.sim(1500, d = 0.3)
spm <- fdSperio(memory.long$series)
str(spm, digits=6)
```

`fracdiff` *ML Estimates for Fractionally-Differenced ARIMA (p,d,q) models*

Description

Calculates the maximum likelihood estimators of the parameters of a fractionally-differenced ARIMA (p,d,q) model, together (if possible) with their estimated covariance and correlation matrices and standard errors, as well as the value of the maximized likelihood. The likelihood is approximated using the fast and accurate method of Haslett and Raftery (1989).

Usage

```
fracdiff(x, nar = 0, nma = 0,
         ar = rep(NA, max(nar, 1)), ma = rep(NA, max(nma, 1)),
         dtol = NULL, drange = c(0, 0.5), h, M = 100, trace = 0)
```

Arguments

`x`	time series (numeric vector) for the ARIMA model
`nar`	number of autoregressive parameters p.
`nma`	number of moving average parameters q.
`ar`	initial autoregressive parameters.
`ma`	initial moving average parameters.
`dtol`	interval of uncertainty for d. If `dtol` is negative or NULL, the fourth root of machine precision will be used. `dtol` will be altered if necessary by the program.
`drange`	interval over which the likelihood function is to be maximized as a function of d.
`h`	size of finite difference interval for numerical derivatives. By default (or if negative), `h = min(0.1, eps.5 * (1+ abs(cllf)))`, where `clff := log. max.likelihood` (as returned) and `eps.5 := sqrt(.Machine$double.neg.eps)` (typically 1.05e-8). This is used to compute a finite difference approximation to the Hessian, and hence only influences the cov, cor, and std.error computations; use `fracdiff.var()` to change this *after* the estimation process.
`M`	number of terms in the likelihood approximation (see Haslett and Raftery 1989).
`trace`	optional integer, specifying a trace level. If positive, currently the "outer loop" iterations produce one line of diagnostic output.

Details

The **fracdiff** package has — for historical reason, namely, S-plus `arima()` compatibility — used an unusual parametrization for the MA part, see also the 'Details' section in `arima` (in standard R's **stats** package). The ARMA (i.e., $d = 0$) model in `fracdiff()` and `fracdiff.sim()` is

$$X_t - a_1 X_{t-1} - \cdots - a_p X_{t-p} = e_t - b_1 e_{t-1} - \ldots - b_q e_{t-q},$$

where e_i are mean zero i.i.d., for `fracdiff()`'s estimation, $e_i \sim \mathcal{N}(0, \sigma^2)$. This model indeed has the signs of the MA coefficients b_j *inverted*, compared to other parametrizations, including Wikipedia's `http://en.wikipedia.org/wiki/Autoregressive_moving-average_model` and the one of `arima`.

Note that `NA`'s in the initial values for `ar` or `ma` are replaced by 0's.

Value

an object of S3 `class` `"fracdiff"`, which is a list with components:

`log.likelihood`	logarithm of the maximum likelihood
`d`	optimal fractional-differencing parameter
`ar`	vector of optimal autoregressive parameters
`ma`	vector of optimal moving average parameters

covariance.dpq covariance matrix of the parameter estimates (order : d, ar, ma).

stderror.dpq standard errors of the parameter estimates c(d,ar,ma).

correlation.dpq
correlation matrix of the parameter estimates (order : d, ar, ma).

h interval used for numerical derivatives, see h argument.

dtol interval of uncertainty for d; possibly altered from input dtol.

M as input.

hessian.dpq the approximate Hessian matrix H of 2nd order partial derivatives of the likelihood with respect to the parameters; this is (internally) used to compute covariance.dpq, the approximate asymptotic covariance matrix as $C = (-H)^{-1}$.

Method

The optimization is carried out in two levels:
an outer univariate unimodal optimization in d over the interval drange (typically [0,.5]), using Brent's fmin algorithm), and
an inner nonlinear least-squares optimization in the AR and MA parameters to minimize white noise variance (uses the MINPACK subroutine lmDER). written by Chris Fraley (March 1991).

Warning

The variance-covariance matrix and consequently the standard errors may be quite inaccurate, see the example in fracdiff.var.

Note

Ordinarily, nar and nma should not be too large (say < 10) to avoid degeneracy in the model. The function fracdiff.sim is available for generating test problems.

References

J. Haslett and A. E. Raftery (1989) Space-time Modelling with Long-memory Dependence: Assessing Ireland's Wind Power Resource (with Discussion); *Applied Statistics* **38**, 1–50.

R. Brent (1973) *Algorithms for Minimization without Derivatives*, Prentice-Hall

J. J. More, B. S. Garbow, and K. E. Hillstrom (1980) *Users Guide for MINPACK-1*, Technical Report ANL-80-74, Applied Mathematics Division, Argonne National Laboratory.

See Also

coef.fracdiff and other methods for "fracdiff" objects; fracdiff.var() for re-estimation of variances or standard errors; fracdiff.sim

Examples

```
ts.test <- fracdiff.sim( 5000, ar = .2, ma = -.4, d = .3)
fd. <- fracdiff( ts.test$series,
                 nar = length(ts.test$ar), nma = length(ts.test$ma))
fd.
## Confidence intervals
confint(fd.)

## with iteration output
fd2 <- fracdiff(ts.test$series, nar = 1, nma = 1, trace = 1)
all.equal(fd., fd2)
```

---

fracdiff-methods — *Many Methods for "fracdiff" Objects*

Description

Many "accessor" methods for `fracdiff` objects, notably `summary`, `coef`, `vcov`, and `logLik`; further `print()` methods were needed.

Usage

```
## S3 method for class 'fracdiff'
coef(object, ...)
## S3 method for class 'fracdiff'
logLik(object, ...)
## S3 method for class 'fracdiff'
print(x, digits = getOption("digits"), ...)
## S3 method for class 'fracdiff'
summary(object, symbolic.cor = FALSE, ...)
## S3 method for class 'summary.fracdiff'
print(x, digits = max(3, getOption("digits") - 3),
        correlation = FALSE, symbolic.cor = x$symbolic.cor,
        signif.stars = getOption("show.signif.stars"), ...)
## S3 method for class 'fracdiff'
fitted(object, ...)
## S3 method for class 'fracdiff'
residuals(object, ...)
## S3 method for class 'fracdiff'
vcov(object, ...)
```

Arguments

`x, object`	object of class `fracdiff`.
`digits`	the number of significant digits to use when printing.
`...`	further arguments passed from and to methods.

`correlation` logical; if TRUE, the correlation matrix of the estimated parameters is returned and printed.

`symbolic.cor` logical. If TRUE, print the correlations in a symbolic form (see symnum) rather than as numbers.

`signif.stars` logical. If TRUE, "significance stars" are printed for each coefficient.

Author(s)

Martin Maechler; Rob Hyndman contributed the residuals() and fitted() methods.

See Also

fracdiff to get "fracdiff" objects, confint.fracdiff for the confint method; further, fracdiff.var.

Examples

```
set.seed(7)
ts4 <- fracdiff.sim(10000, ar = c(0.6, -.05, -0.2), ma = -0.4, d = 0.2)
modFD <- fracdiff( ts4$series, nar = length(ts4$ar), nma = length(ts4$ma))
## -> warning (singular Hessian) %% FIXME ???
coef(modFD) # the estimated parameters
vcov(modFD)
smFD <- summary(modFD)
smFD
coef(smFD) # gives the whole table
AIC(modFD) # AIC works because of the logLik() method
stopifnot(exprs = {

})
```

`fracdiff.sim` *Simulate fractional ARIMA Time Series*

Description

Generates simulated long-memory time series data from the fractional ARIMA(p,d,q) model. This is a test problem generator for fracdiff.

Note that the MA coefficients have *inverted* signs compared to other parametrizations, see the details in fracdiff.

Usage

```
fracdiff.sim(n, ar = NULL, ma = NULL, d,
             rand.gen = rnorm, innov = rand.gen(n+q, ...),
             n.start = NA, backComp = TRUE, allow.0.nstart = FALSE,
             start.innov = rand.gen(n.start, ...),
             ..., mu = 0)
```

Arguments

`n`	length of the time series.
`ar`	vector of autoregressive parameters; empty by default.
`ma`	vector of moving average parameters; empty by default.
`d`	fractional differencing parameter.
`rand.gen`	a function to generate the innovations; the default, `rnorm` generates white N(0,1) noise.
`innov`	an optional times series of innovations. If not provided, `rand.gen()` is used.
`n.start`	length of "burn-in" period. If `NA`, the default, the same value as in `arima.sim` is computed.
`backComp`	logical indicating if back compatibility with older versions of `fracdiff.sim` is desired. Otherwise, for `d = 0`, compatibility with R's `arima.sim` is achieved.
`allow.0.nstart`	logical indicating if `n.start = 0` should be allowed even when $p + q > 0$. This not recommended unless for producing the same series as with older versions of `fracdiff.sim`.
`start.innov`	an optional vector of innovations to be used for the burn-in period. If supplied there must be at least `n.start` values.
`...`	additional arguments for `rand.gen()`. Most usefully, the standard deviation of the innovations generated by `rnorm` can be specified by `sd`.
`mu`	time series mean (added at the end).

Value

a list containing the following elements :

`series`	time series
`ar, ma, d, mu, n.start`	same as input

See Also

`fracdiff`, also for references; `arima.sim`

Examples

```
## Pretty (too) short to "see" the long memory
fracdiff.sim(100, ar = .2, ma = .4, d = .3)

## longer with "extreme" ar:
r <- fracdiff.sim(n=1500, ar=-0.9, d= 0.3)
plot(as.ts(r$series))

## Show that  MA  coefficients meaning is inverted
## compared to   stats :: arima :

AR <- 0.7
```

```
MA <- -0.5
n.st <- 2

AR <- c(0.7, -0.1)
MA <- c(-0.5, 0.4)
n <- 512 ; sd <- 0.1
n.st <- 10

set.seed(101)
Y1 <- arima.sim(list(ar = AR, ma = MA), n = n, n.start = n.st, sd = sd)
plot(Y1)

# For our fracdiff, reverse the MA sign:
set.seed(101)
Y2 <- fracdiff.sim(n = n, ar = AR, ma = - MA, d = 0,
                   n.start = n.st, sd = sd)$series
lines(Y2, col=adjustcolor("red", 0.5))
## .. no, you don't need glasses ;-)  Y2 is Y1 shifted slightly

##' rotate left by k (k < 0: rotate right)
rot <- function(x, k) {
  stopifnot(k == round(k))
  n <- length(x)
  if(k <- k %% n) x[c((k+1):n, 1:k)] else x
}
k <- n.st - 2
Y2.s <- rot(Y2, k)
head.matrix(cbind(Y1, Y2.s))
plot(Y1, Y2.s); i <- (n-k+1):n
text(Y1[i], Y2.s[i], i, adj = c(0,0)-.1, col=2)

## With  backComp = FALSE,  get *the same* as arima.sim():
set.seed(101)
Y2. <- fracdiff.sim(n = n, ar = AR, ma = - MA, d = 0,
                    n.start = n.st, sd = sd, backComp = FALSE)$series
stopifnot( all.equal( c(Y1), Y2., tolerance= 1e-15))
```

`fracdiff.var` *Recompute Covariance Estimate for fracdiff*

Description

Allows the finite-difference interval to be altered for recomputation of the covariance estimate for `fracdiff`.

Usage

```
fracdiff.var(x, fracdiff.out, h)
```

Arguments

x	a univariate time series or a vector. Missing values (NAs) are not allowed.
fracdiff.out	output from fracdiff for time series x.
h	finite-difference interval for approximating partial derivatives with respect to the d parameter.

Value

an object of S3 class "fracdiff", i.e., basically a list with the same elements as the result from fracdiff, but with possibly different values for the hessian, covariance, and correlation matrices and for standard error, as well as for h.

See Also

fracdiff, also for references.

Examples

```
## Generate a fractionally-differenced ARIMA(1,d,1) model :
ts.test <- fracdiff.sim(10000, ar = .2, ma = .4, d = .3)
## estimate the parameters in an ARIMA(1,d,1) model for the simulated series
fd.out <- fracdiff(ts.test$ser, nar= 1, nma = 1)

## Modify the covariance estimate by changing the finite-difference interval
(fd.o2 <- fracdiff.var(ts.test$series, fd.out, h = .0001))
## looks identical as  print(fd.out),
## however these (e.g.) differ :
vcov(fd.out)
vcov(fd.o2)

## A case, were the default variance is *clearly* way too small:
set.seed(1); fdc <- fracdiff(X <- fracdiff.sim(n=100,d=0.25)$series)
fdc
# Confidence intervals just based on asymp.normal approx. and std.errors:
confint(fdc) # ridiculously too narrow
```

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