# Package 'heplots' 

October 6, 2021

## Type Package

Title Visualizing Hypothesis Tests in Multivariate Linear Models
Version 1.3-9
Date 2021-10-06
Author John Fox [aut],
Michael Friendly [aut, cre],
Georges Monette [ctb],
Phil Chalmers [ctb]
Maintainer Michael Friendly [friendly@yorku.ca](mailto:friendly@yorku.ca)
Language en-US
Depends R (>= 3.5.0), car (>= 3.0)
Imports MASS, graphics, grDevices, stats, utils
Suggests rgl, candisc, effects, reshape, gplots, nlme, lattice, reshape2, corrgram, animation, mvinfluence
LazyLoad yes
LazyData yes
BugReports https://github.com/friendly/heplots/issues
Description Provides HE plot and other functions for visualizing hypothesis tests in multivariate linear models. HE plots represent sums-of-squares-and-products matrices for linear hypotheses and for error using ellipses (in two dimensions) and ellipsoids (in three dimensions). The related 'candisc' package provides visualizations in a reduced-rank canonical discriminant space when there are more than a few response variables.

License GPL (>= 2)
URL http://friendly.github.io/heplots/
RoxygenNote 5.0.1
NeedsCompilation no
Repository CRAN
Date/Publication 2021-10-06 21:00:02 UTC

## $R$ topics documented:

heplots-package ..... 3
AddHealth ..... 5
Adopted ..... 6
arrow3d ..... 8
bartlettTests ..... 9
Bees ..... 10
boxM ..... 12
coefplot ..... 14
colDevs ..... 16
covEllipses ..... 18
cqplot ..... 21
cross3d ..... 24
Diabetes ..... 25
ellipse3d.axes ..... 27
etasq ..... 28
FootHead ..... 30
gsorth ..... 31
Headache ..... 33
heplot ..... 35
heplot1d ..... 41
heplot3d ..... 44
Hernior ..... 48
interpPlot ..... 50
label.ellipse ..... 53
leveneTests ..... 55
logdetCI ..... 56
Mahalanobis ..... 58
mark.H0 ..... 59
mathscore ..... 60
MockJury ..... 61
NeuroCog ..... 62
NLSY ..... 64
Oslo ..... 66
pairs.mlm ..... 67
Parenting ..... 69
Plastic ..... 71
plot.boxM ..... 72
plot.robmlm ..... 73
Pottery2 ..... 75
Probe ..... 77
RatWeight ..... 79
ReactTime ..... 80
robmlm ..... 81
Rohwer ..... 85
RootStock ..... 87
Sake ..... 88
schooldata ..... 89
Skulls ..... 90
SocGrades ..... 92
SocialCog ..... 94
statList . ..... 95
termMeans ..... 97
trans.colors ..... 98
VocabGrowth ..... 99
WeightLoss ..... 100
Index ..... 104heplots-package Visualizing Hypothesis Tests in Multivariate Linear Models

## Description

The heplots package provides functions for visualizing hypothesis tests in multivariate linear models (MANOVA, multivariate multiple regression, MANCOVA, and repeated measures designs). HE plots represent sums-of-squares-and-products matrices for linear hypotheses and for error using ellipses (in two dimensions), ellipsoids (in three dimensions), or by line segments in one dimension. See Fox, Friendly and Monette (2007) for a brief introduction and Friendly, Monette and Fox (2013) for a general discussion of the role of elliptical geometry in statistical understanding.
Other topics now addressed here include robust MLMs, tests for equality of covariance matrices in MLMs, and chi square Q-Q plots for MLMs.
The package also provides a collection of data sets illustrating a variety of multivariate linear models of the types listed above, together with graphical displays.
Several tutorial vignettes are also included. See vignette(package="heplots").

## Details

Package: heplots
Type: Package
Version: 1.3-9
Date: 2021-10-06
License: GPL (>=2), GPL version 2 or newer

The graphical functions contained here all display multivariate model effects in variable (data) space, for one or more response variables (or contrasts among response variables in repeated measures designs).
heplot constructs two-dimensional HE plots for model terms and linear hypotheses for pairs of response variables in multivariate linear models.
heplot3d constructs analogous 3D plots for triples of response variables.
pairs.mlm constructs a "matrix" of pairwise HE plots.
heplot1d constructs 1-dimensional analogs of HE plots for model terms and linear hypotheses for single response variables.

For repeated measure designs, between-subject effects and within-subject effects must be plotted separately, because the error terms (E matrices) differ. For terms involving within-subject effects, these functions carry out a linear transformation of the matrix $\mathbf{Y}$ of responses to a matrix $\mathbf{Y} \mathbf{M}$, where $\mathbf{M}$ is the model matrix for a term in the intra-subject design and produce plots of the H and E matrices in this transformed space. The vignette repeated describes these graphical methods for repeated measures designs.
The related car package calculates Type II and Type III tests of multivariate linear hypotheses using the Anova and linearHypothesis functions.

The candisc-package package provides functions for visualizing effects for MLM model terms in a low-dimensional canonical space that shows the largest hypothesis relative to error variation. The candisc package now also includes related methods for canonical correlation analysis.
The heplots package also contains a large number of multivariate data sets with examples of analyses and graphic displays. Use data(package="heplots") to see the current list.

## Author(s)

John Fox, Michael Friendly, and Georges Monette
Maintainer: Michael Friendly, [friendly@yorku.ca](mailto:friendly@yorku.ca), http://datavis.ca

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples. Journal of Statistical Software, 17(6), 1-42.
doi: 10.18637/jss.v017.i06
Friendly, M. (2007). HE plots for Multivariate General Linear Models. Journal of Computational and Graphical Statistics, 16(2) 421-444. http: //datavis.ca/papers/jcgs-heplots.pdf
Fox, J., Friendly, M. \& Monette, G. (2007). Visual hypothesis tests in multivariate linear models: The heplots package for R. DSC 2007: Directions in Statistical Computing. https://socialsciences. mcmaster.ca/jfox/heplots-dsc-paper.pdf
Friendly, M. (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04
Fox, J., Friendly, M. \& Weisberg, S. (2013). Hypothesis Tests for Multivariate Linear Models Using the car Package. The R Journal, 5(1), https://journal.r-project.org/archive/2013-1/ fox-friendly-weisberg.pdf.

Friendly, M., Monette, G. \& Fox, J. (2013). Elliptical Insights: Understanding Statistical Methods Through Elliptical Geometry. Statistical Science, 2013, 28 (1), 1-39, http://datavis.ca/ papers/ellipses.pdf.
Friendly, M. \& Sigal, M. (2014). Recent Advances in Visualizing Multivariate Linear Models. Revista Colombiana de Estadistica, 37, 261-283
Friendly, M. \& Sigal, M. (2016). Graphical Methods for Multivariate Linear Models in Psychological Research: An R Tutorial. Submitted for publication.

## See Also

Anova, linearHypothesis for Anova.mlm computations and tests
candisc-package for reduced-rank views in canonical space
manova for a different approach to testing effects in MANOVA designs

## AddHealth <br> Adolescent Health Data

## Description

This data was taken from the National Longitudinal Study of Adolescent Health. It is a crosssectional sample of participants from grades 7-12, described and analyzed by Warne (2014).

## Usage

```
data("AddHealth")
```


## Format

A data frame with 4344 observations on the following 3 variables.
grade an ordered factor with levels $7<8<9<10<11<12$
depression a numeric vector
anxiety a numeric vector

## Details

depression is the response to the question "In the last month, how often did you feel depressed or blue?"
anxiety is the response to the question "In the last month, how often did you have trouble relaxing?"
The responses for depression and anxiety were recorded on a 5-point Likert scale, with categories $0=" N e v e r ", 1=" R a r e l y ", 2=" O c c a s i o n a l l y ", 3=" O f t e n ", 4=" E v e r y ~ d a y "$

## Source

Warne, R. T. (2014). A primer on Multivariate Analysis of Variance (MANOVA) for Behavioral Scientists. Practical Assessment, Research \& Evaluation, 19 (1). https://scholarworks.umass. edu/pare/vol19/iss1/17/

## Examples

```
data(AddHealth)
# fit mlm
AH.mod <- lm(cbind(depression, anxiety) ~ grade, data=AddHealth)
Anova(AH.mod)
summary(Anova(AH.mod))
heplot(AH.mod, hypotheses="grade.L", fill=c(TRUE, FALSE))
```

Adopted Adopted Children

## Description

Data are a subset from an observational, longitudinal, study on adopted children. Is child's intelligence related to intelligence of the biological mother and the intelligence of the adoptive mother?

The child's intelligence was measured at age $2,4,8$, and 13 for this sample. How does intelligence change over time, and how are these changes related to intelligence of the birth and adoptive mother?

## Usage

Adopted

## Format

A data frame with 62 observations on the following 6 variables.
AMED adoptive mother's years of education (proxy for her IQ)
BMIQ biological mother's score on IQ test
Age2IQ IQ of child at age 2
Age4IQ IQ of child at age 4
Age8IQ IQ of child at age 8
Age13IQ IQ of child at age 13

## Source

Ramsey, F.L. and Schafer, D.W. (2002). The Statistical Sleuth: A Course in Methods of Data Analysis (2nd ed), Duxbury.

This data set is identical to ex1605 in the Sleuth2 package.

## References

Friendly, Michael (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40. doi: 10.18637/jss.v037.i04.

Skodak, M. and Skeels, H.M. (1949). A Final Follow-up Study of One Hundred Adopted Children, Journal of Genetic Psychology 75: 85-125.

## See Also

ex1605

## Examples

```
# Treat as multivariate regression problem
Adopted.mod <- lm(cbind(Age2IQ, Age4IQ, Age8IQ, Age13IQ) ~ AMED + BMIQ, data=Adopted)
Adopted.mod
require(car)
# test overall multivariate regression
linearHypothesis(Adopted.mod, c("AMED","BMIQ"))
# show separate linear regressions
op <- par(mfcol=c(2,4), mar=c(4,4,2,2)+.1)
for (i in 3:6) {
dataEllipse(as.matrix(Adopted[,c(1,i)]),col="black", levels=0.68, ylim=c(70,140))
abline(lm(Adopted[,i] ~ Adopted[,1]), col="red", lwd=2)
dataEllipse(as.matrix(Adopted[,c(2,i)]),col="black", levels=0.68, ylim=c(70,140))
abline(lm(Adopted[,i] ~ Adopted[,2]), col="red", lwd=2)
abline(a=0,b=1, lty=1, col="blue")
}
par(op)
# between-S (MMReg) plots
heplot(Adopted.mod, hypotheses=list("Reg"=c("AMED", "BMIQ")),
main="IQ scores of adopted children: MMReg")
pairs(Adopted.mod, hypotheses=list("Reg"=c("AMED", "BMIQ")))
if(requireNamespace("rgl")){
heplot3d(Adopted.mod, hypotheses=list("Reg"=c("AMED", "BMIQ")),
col = c("red", "blue", "black", "gray"), wire=FALSE)
}
# Treat IQ at different ages as a repeated measure factor
# within-S models & plots
Age <- data.frame(Age=ordered(c(2,4,8,13)))
Anova(Adopted.mod, idata=Age, idesign=~Age, test="Roy")
# within-S plots
heplot(Adopted.mod, idata=Age, idesign=~Age, iterm="Age",
cex=1.25, cex.lab=1.4, fill=c(FALSE, TRUE),
hypotheses=list("Reg"=c("AMED", "BMIQ"))
```

)
arrow3d
Draw a 3D Arrow in an RGL Scene

## Description

Draws a 3D arrow in an rgl scene with barbs at the arrow head

## Usage

$\operatorname{arrow} 3 \mathrm{~d}(\mathrm{p} 0=\mathrm{c}(0,0,0), \mathrm{p} 1=\mathrm{c}(1,1,1)$, barblen, $\mathrm{s}=0.05$, theta $=\mathrm{pi} / 6, \mathrm{n}=3, \ldots)$

## Arguments

| p 0 | Initial point (tail of arrow) |
| :--- | :--- |
| p 1 | Ending point (head of arrow) |
| barblen | Length of each barb, in data units |
| s | length of barb as fraction of line length (unless barblen is specified) |
| theta | opening angle of barbs |
| n | number of barbs |
| $\ldots$. | args passed to lines3d for line styling, e.g., color, lwd, etc. See material3d. |

## Value

Returns (invisibly): integer ID of the line added to the scene

## Author(s)

Barry Rowlingson, posted to R-help, 1/10/2010

## See Also

lines3d, segments3d,

## Examples

```
arrow3d(c(0,0,0), c(2,2,2), barblen=.2, lwd=3, col="black")
arrow3d(c(0,0,0), c(-2,2,2), barblen=.2, lwd=3, col="red")
```

```
bartlettTests Bartlett Tests of Homogeneity of Variances
```


## Description

This function extends bartlett. test to a multivariate response setting. It performs the Bartlett test of homogeneity of variances for each of a set of response variables, and prints a compact summary.

## Usage <br> bartlettTests(y, group, ...)

## Arguments

$y \quad$ A data frame or matrix of numeric response variables in a multivariate linear model.
group a vector or factor object giving the group for the corresponding elements of the rows of $y$
... other arguments, passed to bartlett. test

## Details

Bartlett's test is the univariate version of Box's $M$ test for equality of covariance matrices. This function provides a univariate follow-up test to Box's $M$ test to give one simple assessment of which response variables contribute to significant differences in variances among groups.

## Value

An object of classes "anova" and "data.frame", with one observation for each response variable in y .

## Author(s)

Michael Friendly

## References

Bartlett, M. S. (1937). Properties of sufficiency and statistical tests. Proceedings of the Royal Society of London Series A, 160, 268-282.

## See Also

boxM for Box's M test for all responses.

## Examples

```
bartlettTests(iris[,1:4], iris$Species)
data(Skulls, package="heplots")
bartlettTests(Skulls[,-1], Skulls$epoch)
```

Bees Captive and maltreated bees

## Description

Pabalan, Davey and Packe (2000) studied the effects of captivity and maltreatment on reproductive capabilities of queen and worker bees in a complex factorial design.
Bees were placed in a small tube and either held captive (CAP) or shaken periodically (MAL) for one of $5,7.5,10,12.5$ or 15 minutes, after which they were sacrificed and two measures: ovarian development (Iz) and ovarian reabsorption (Iy), were taken. A single control group was measured with no such treatment, i.e., at time 0 ; there are $\mathrm{n}=10$ per group.
The design is thus nearly a three-way factorial, with factors caste (Queen, Worker), treat (CAP, MAL) and time, except that there are only 11 combinations of Treatment and Time; we call these trtime below.

## Usage

data(Bees)

## Format

A data frame with 246 observations on the following 6 variables.
caste a factor with levels Queen Worker
treat a factor with levels "" CAP MAL
time an ordered factor: time of treatment
Iz an index of ovarian development
Iy an index of ovarian reabsorption
trtime a factor with levels 0 CAP05 CAP07 CAP10 CAP12 CAP15 MAL05 MAL07 MAL10 MAL12 MAL15

## Details

Models for the three-way factorial design, using the formula cbind(Iz, Iy) ~ caste*treat*time ignore the control condition at time $==0$, where treat==NA.
To handle the additional control group at time==0, while separating the effects of Treatment and Time, 10 contrasts can be defined for the trtime factor in the model cbind(Iz, Iy) ~ caste*trtime See demo(bees.contrasts) for details.
In the heplot examples below, the default size="evidence" displays are too crowded to interpret, because some effects are so highly significant. The alternative effect-size scaling, size="effect", makes the relations clearer.

## Source

Pabalan, N., Davey, K. G. \& Packe, L. (2000). Escalation of Aggressive Interactions During Staged Encounters in Halictus ligatus Say (Hymenoptera: Halictidae), with a Comparison of Circle Tube Behaviors with Other Halictine Species Journal of Insect Behavior, 13, 627-650.

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples Journal of Statistical Software, 17, 1-42.

## Examples

```
data(Bees)
require(car)
# 3-way factorial, ignoring 0 group
bees.mod <- lm(cbind(Iz,Iy) ~ caste*treat*time, data=Bees)
Anova(bees.mod)
op<-palette(c(palette()[1:4],"brown","magenta", "olivedrab","darkgray"))
heplot(bees.mod, xlab="Iz: Ovarian development", ylab="Iz: Ovarian reabsorption",
main="Bees: ~caste*treat*time")
heplot(bees.mod, xlab="Iz: Ovarian development", ylab="Iz: Ovarian reabsorption",
    main="Bees: ~caste*treat*time", size="effect")
# two-way design, using trtime
bees.mod1 <- lm(cbind(Iz,Iy) ~ caste*trtime, data=Bees)
Anova(bees.mod1)
heplot(bees.mod1, xlab="Iz: Ovarian development", ylab="Iz: Ovarian reabsorption",
main="Bees: ~caste*trtime")
heplot(bees.mod1, xlab="Iz: Ovarian development", ylab="Iz: Ovarian reabsorption",
    main="Bees: ~caste*trtime",size="effect")
palette(op)
# effect plots for separate responses
if(require(effects)) {
bees.lm1 <-lm(Iy ~ treat*caste*time, data=Bees)
bees.lm2 <-lm(Iz ~ treat*caste*time, data=Bees)
bees.eff1 <- allEffects(bees.lm1)
plot(bees.eff1,multiline=TRUE,ask=FALSE)
bees.eff2 <- allEffects(bees.lm2)
plot(bees.eff2,multiline=TRUE,ask=FALSE)
}
```

boxM Box's M-test

## Description

boxM performs the Box's (1949) M-test for homogeneity of covariance matrices obtained from multivariate normal data according to one or more classification factors. The test compares the product of the $\log$ determinants of the separate covariance matrices to the log determinant of the pooled covariance matrix, analogous to a likelihood ratio test. The test statistic uses a chi-square approximation.

## Usage

```
boxM(Y, ...)
## S3 method for class 'formula'
boxM(Y, data, ...)
## S3 method for class 'lm'
boxM(Y, ...)
## Default S3 method:
boxM(Y, group, ...)
## S3 method for class 'boxM'
summary(object,
    digits = getOption("digits"),
    cov=FALSE, quiet=FALSE, ...)
```


## Arguments

| $Y$ | The response variable matrix for the default method, or a "mlm" or "formula" <br> object for a multivariate linear model. If $Y$ is a linear-model object or a formula, <br> the variables on the right-hand-side of the model must all be factors and must be <br> completely crossed, e.g., A: B |
| :--- | :--- |
| data | a numeric data.frame or matrix containing $n$ observations of $p$ variables; it is <br> expected that $n>p$. |
| group | a factor defining groups, or a vector of length $n$ doing the same. |
| object | a "boxM" object for the summary method |
| digits | number of digits to print for the summary method |
| cov quiet | logical; if TRUE the covariance matrices are printed. |
| $\ldots$ | logical; if TRUE printing from the summary is suppressed |

## Details

As an object of class "htest", the statistical test is printed normally by default. As an object of class "boxM", a few methods are available.
There is no general provision as yet for handling missing data. Missing data are simply removed, with a warning.

As well, the computation assumes that the covariance matrix for each group is non-singular, so that $\log \operatorname{det}\left(S_{i}\right)$ can be calculated for each group. At the minimum, this requires that $n>p$ for each group.
Box's M test for a multivariate linear model highly sensitive to departures from multivariate normality, just as the analogous univariate test. It is also affected adversely by unbalanced designs. Some people recommend to ignore the result unless it is very highly significant, e.g., p $<.0001$ or worse.
The summary method prints a variety of additional statistics based on the eigenvalues of the covariance matrices. These are returned invisibly, as a list containing the following components:

- logDet - log determinants
- eigs - eigenvalues of the covariance matrices
- eigstats - statistics computed on the eigenvalues for each covariance matrix: product: the product of eigenvalues, $\prod \lambda_{i}$; sum: the sum of eigenvalues, $\sum \lambda_{i}$; precision: the average precision of eigenvalues, $1 / \sum\left(1 / \lambda_{i}\right)$; max: the maximum eigenvalue, $\lambda_{1}$


## Value

A list with class c("htest", "boxM") containing the following components:

| statistic | an approximated value of the chi-square distribution. |
| :--- | :--- |
| parameter |  |
| the degrees of freedom related of the test statistic in this case that it follows a |  |
| Chi-square distribution. |  |
| p.value | the p-value of the test. <br> a list containing the within covariance matrix for each level of grouping. |
| cov | the pooled covariance matrix. |
| pooled | a vector containing the natural logarithm of each matrix in cov, followed by the <br> value for the pooled covariance matrix |
| means | a matrix of the means for all groups, followed by the grand means <br> a vector of the degrees of freedom for all groups, followed by that for the pooled <br> covariance matrix |
| df | a character string giving the names of the data. |
| data. name character string "Box's M-test for Homogeneity of Covariance Matrices". |  |

## Author(s)

The default method was taken from the biotools package, Anderson Rodrigo da Silva <anderson. agro@hotmail.com>
Generalized by Michael Friendly and John Fox

## References

Box, G. E. P. (1949). A general distribution theory for a class of likelihood criteria. Biometrika, 36, 317-346.

Morrison, D.F. (1976) Multivariate Statistical Methods.

## See Also

leveneTest carries out homogeneity of variance tests for univariate models with better statistical properties.
plot.boxM, a simple plot of the $\log$ determinants
covEllipses plots covariance ellipses in variable space for several groups.

## Examples

```
data(iris)
# default method
res <- boxM(iris[, 1:4], iris[, "Species"])
res
summary(res)
# visualize (what is done in the plot method)
dets <- res$logDet
ng <- length(res$logDet)-1
dotchart(dets, xlab = "log determinant")
points(dets , 1:4,
cex=c(rep(1.5, ng), 2.5),
pch=c(rep(16, ng), 15),
col= c(rep("blue", ng), "red"))
# formula method
boxM( cbind(Sepal.Length, Sepal.Width, Petal.Length, Petal.Width) ~ Species, data=iris)
### Skulls dat
data(Skulls)
# lm method
skulls.mod <- lm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
boxM(skulls.mod)
```


## Description

Displays confidence ellipses for all parameters in an multivariate linear model, for a given pair of variables. As such, it is a generalization of confidenceEllipse.

## Usage

```
coefplot(object, ...)
## S3 method for class 'mlm'
coefplot(object, variables = 1:2, parm = NULL,
df = NULL, level = 0.95, intercept = FALSE,
Scheffe = FALSE, bars = TRUE,
fill = FALSE, fill.alpha = 0.2,
labels = !add, label.pos = NULL,
xlab, ylab,
xlim = NULL, ylim = NULL,
axes = TRUE, main = "", add = FALSE,
lwd = 1, lty = 1, pch = 19, col = palette(),
cex = 2, cex.label = 1.5,
lty.zero = 3, col.zero = 1, pch.zero = '+',
verbose = FALSE, ...)
```


## Arguments

| object | A multivariate linear model, such as fit by $\operatorname{lm}()$ |
| :---: | :---: |
|  | Other parameters passed to methods |
| variables | Response variables to plot, given as their indices or names |
| parm | Parameters to plot |
| df | Degrees of freedom for hypothesis tests |
| level | Confidence level |
| intercept | logical. Include the intercept? |
| Scheffe | If TRUE, confidence intervals for all parameters have Scheffe coverage, otherwise, individual coverage. |
| bars | Draw univariate confidence intervals for each of the variables? |
| fill | a logical value or vector. TRUE means the confidence ellipses will be filled. |
| fill.alpha |  |
| labels | Labels for the confidence ellipses |
| label.pos | Positions of the labels for each ellipse. See label.ellipse |
| xlab, ylab | $\mathrm{x}, \mathrm{y}$ axis labels |
| xlim, ylim | Axis limits |
| axes | Draw axes? |
| main | Plot title |


| add | logical. Add to an existing plot? |
| :--- | :--- |
| lwd | Line widths |
| $l$ ty | Line types |
| pch | Point symbols for the parameter estimates |
| col | Colors for the confidence ellipses, points, lines |
| cex | Character size for points showing parameter estimates |
| cex.label | Character size for ellipse labels |
| $l$ ly.zero, col.zero, pch.zero |  |
|  | Line type, color and point symbol for horizontal and vertical lines at $0,0$. |
| verbose | logical. Print parameter estimates and variance-covariance for each parameter? |

## Value

Returns invisibly a list of the coordinates of the ellipses drawn

## Author(s)

Michael Friendly

## See Also

confidenceEllipse, ~~~

## Examples

```
mod <- lm(cbind(SAT,PPVT,Raven)~n+s+ns, data=Rohwer)
coefplot(mod, lwd=2, main="Bivariate coefficient plot for SAT and PPVT", fill=TRUE)
coefplot(mod, add=TRUE, Scheffe=TRUE, fill=TRUE)
coefplot(mod, var=c(1,3))
mod1 <- lm(cbind(SAT,PPVT,Raven)~n+s+ns+na+ss, data=Rohwer)
coefplot(mod1, lwd=2, fill=2 !=(1:5),
main="Bivariate 68% coefficient plot for SAT and PPVT", level=0.68)
```

```
colDevs Calculate column deviations from central values
```


## Description

colDevs calculates the column deviations of data values from a central value (mean, median, etc.), possibly stratified by a grouping variable.

Conceptually, the function is similar to a column-wise sweep, by group, allowing an arbitrary center function.

## Usage

colDevs(x, group, center = mean, ...)

## Arguments

## X

A numeric data frame or matrix.
group A factor (or variable that can be coerced to a factor) indicating the membership of each observation in $x$ in one or more groups. If missing, all the data is treated as a single group.
center A function used to center the values (for each group if group is specified. The function must take a vector argument and return a scalar result.
... Arguments passed down

## Details

Non-numeric columns of x are removed, with a warning.

## Value

A numeric matrix containing the deviations from the centering function.

## Author(s)

Michael Friendly

## See Also

colMeans for column means,
sweep

## Examples

```
data(iris)
Species <- iris$Species
irisdev <- colDevs(iris[,1:4], Species, mean)
irisdev <- colDevs(iris[,1:4], Species, median)
# trimmed mean, using an anonymous function
irisdev <- colDevs(iris[,1:4], Species, function(x) mean(x, trim=0.25))
# no grouping variable: deviations from column grand means
# include all variables (but suppress warning for this doc)
irisdev <- suppressWarnings( colDevs(iris) )
```


## Description

The function draws covariance ellipses for one or more groups and optionally for the pooled total sample. It uses either the classical product-moment covariance estimate, or a robust alternative, as provided by cov. rob. Provisions are provided to do this for more than two variables, in a scatterplot matrix format.

## Usage

```
covEllipses(x, ...)
## S3 method for class 'boxM'
covEllipses(x, ...)
## S3 method for class 'data.frame'
covEllipses(x, group, pooled = TRUE, method = c("classical", "mve", "mcd"), ...)
## S3 method for class 'matrix'
covEllipses(x, group, pooled = TRUE, method = c("classical", "mve", "mcd"), ...)
## Default S3 method:
covEllipses(x, means, df,
    labels = NULL, variables = 1:2, level = 0.68,
    segments = 60,
    center = FALSE, center.pch = "+", center.cex = 2,
    col = getOption("heplot.colors", c("red", "blue", "black", "darkgreen",
                            "darkcyan", "brown", "magenta", "darkgray")),
lty = 1, lwd = 2,
fill = FALSE, fill.alpha = 0.3,
label.pos = 0,
xlab, ylab,
vlabels, var.cex=2,
main = "",
xlim, ylim, axes = TRUE,
offset.axes, add = FALSE, ...)
```


## Arguments

x
The generic argument. For the default method, this is a list of covariance matrices. For the data.frame and matrix methods, this is a numeric matrix of two or more columns supplying the variables to be analyzed.
group a factor defining groups, or a vector of length $n=n r o w(x)$ doing the same. If missing, a single covariance ellipse is drawn.

| pooled | Logical; if TRUE, the pooled covariance matrix for the total sample is also com- |
| :--- | :--- |
| puted and plotted |  |
| the covariance method to be used: classical product-moment ("classical"), |  |
| or minimum volume ellipsoid ("mve"), or minimum covariance determinant |  |
| ("mcd"). |  |
| means | For the default method, a matrix of the means for all groups (followed by the <br> grand means, if pooled=TRUE). Rows are the groups, and columns are the vari- <br> ables. It is assumed that the means have column names corresponding to the <br> variables in the covariance matrices. |
| df | For the default method, a vector of the degrees of freedom for the covariance <br> matrices |
| labels | Either a character vector of labels for the groups, or TRUE, indicating that group <br> labels are taken as the names of the covariance matrices. Use labels="" to |
| suppress group labels, e.g., when add=TRUE |  |

the max/min coordinates of the ellipse; the value 0 specifies the centroid of the ellipse object. The default, label. pos=NULL uses the correlation of the ellipse to determine "top" ( $\mathrm{r}>=0$ ) or "bottom" ( $\mathrm{r}<0$ ).
$x l a b \quad x$-axis label; defaults to name of the $x$ variable.
ylab $\quad y$-axis label; defaults to name of the $y$ variable.
vlabels Labels for the variables can also be supplied through this argument, which is more convenient when length (variables) $>2$.
var.cex character size for variable labels in the pairs plot
main main plot label; defaults to "", and presently has no effect.
$x \lim \quad x$-axis limits; if absent, will be computed from the data.
$y l i m \quad y$-axis limits; if absent, will be computed from the data.
axes Whether to draw the x , y axes; defaults to TRUE
offset.axes proportion to extend the axes in each direction if computed from the data; optional.
add if TRUE, add to the current plot; the default is FALSE. This argument is has no effect when more than two variables are plotted.
... Other arguments passed to the default method for plot, text, and points

## Details

These plot methods provide one way to visualize possible heterogeneity of within-group covariance matrices in a one-way MANOVA design. When covariance matrices are nearly equal, their covariance ellipses should all have the same shape. When centered at a common mean, they should also all overlap.
The can also be used to visualize the difference between classical and robust covariance matrices.

## Value

Nothing is returned. The function is used for its side-effect of producing a plot.

## Author(s)

Michael Friendly

## See Also

heplot, boxM,
cov.rob

## Examples

```
data(iris)
# compare classical and robust covariance estimates
covEllipses(iris[,1:4], iris$Species)
```

```
covEllipses(iris[,1:4], iris$Species, fill=TRUE, method="mve", add=TRUE, labels="")
# method for a boxM object
x <- boxM(iris[, 1:4], iris[, "Species"])
x
covEllipses(x, fill=c(rep(FALSE,3), TRUE) )
covEllipses(x, fill=c(rep(FALSE,3), TRUE), center=TRUE, label.pos=1:4 )
# method for a list of covariance matrices
cov <- c(x$cov, pooled=list(x$pooled))
df <- c(table(iris$Species)-1, nrow(iris)-3)
covEllipses(cov, x$means, df, label.pos=3, fill=c(rep(FALSE,3), TRUE))
covEllipses(cov, x$means, df, label.pos=3, fill=c(rep(FALSE,3), TRUE), center=TRUE)
# scatterplot matrix version
covEllipses(iris[,1:4], iris$Species,
fill=c(rep(FALSE,3), TRUE), variables=1:4,
fill.alpha=.1)
```

cqplot Chi Square Quantile-Quantile plots

## Description

A chi square quantile-quantile plots show the relationship between data-based values which should be distributed as $\chi^{2}$ and corresponding quantiles from the $\chi^{2}$ distribution. In multivariate analyses, this is often used both to assess multivariate normality and check for outliers, using the Mahalanobis squared distances ( $D^{2}$ ) of observations from the centroid.
cqplot is a more general version of similar functions in other packages that produce chi square QQ plots. It allows for classical Mahalanobis squared distances as well as robust estimates based on the MVE and MCD; it provides an approximate confidence (concentration) envelope around the line of unit slope, a detrended version, where the reference line is horizontal, the ability to identify or label unusual points, and other graphical features.
The method for "mlm" objects applies this to the residuals from the model.

## Usage

```
cqplot(x, ...)
## S3 method for class 'mlm'
cqplot(x, ...)
## Default S3 method:
cqplot(x, method = c("classical", "mcd", "mve"),
detrend = FALSE, pch = 19, col = palette()[1], cex = par("cex"),
```

```
ref.col = "red", ref.lwd = 2,
conf = 0.95, env.col = "gray", env.lwd = 2, env.lty = 1,
env.fill = TRUE, fill.alpha = 0.2,
fill.color = trans.colors(ref.col, fill.alpha),
labels = if (!is.null(rownames(x))) rownames(x) else 1:nrow(x),
id.n, id.method = "y", id.cex = 1, id.col = palette()[1],
xlab, ylab, main, what=deparse(substitute(x)), ylim, ...)
```


## Arguments

x
... Other arguments passed to methods
method estimation method used for center and covariance, one of: "classical" (productmoment), "mcd" (minimum covariance determinant), or "mve" (minimum volume ellipsoid).
detrend logical; if FALSE, the plot shows values of $D^{2}$ vs. $\chi^{2}$. if TRUE, the ordinate shows values of $D^{2}-\chi^{2}$
pch plot symbol for points Can be a vector of length equal to the number of rows in x .
col color for points; the default is the first entry in the current color palette (see palette and par.
cex character symbol size for points. Can be a vector of length equal to the number of rows in $x$.
ref.col Color for the reference line
ref.lwd Line width for the reference line
conf
env.col line color for the boundary of the confidence envelope
env.lwd line width for the confidence envelope
env.lty line type for the confidence envelope
env.fill logical; should the confidence envelope be filled?
fill.alpha transparency value for fill.color
fill.color color used to fill the confidence envelope
labels vector of text strings to be used to identify points, defaults to rownames $(x)$ or observation numbers if rownames ( $x$ ) is NULL
id.n number of points labeled. If id. $n=0$, the default, no point identification occurs.
id.method point identification method. The default id.method="y" will identify the id.n points with the largest value of $\operatorname{abs}(\mathrm{y}-\mathrm{mean}(\mathrm{y}))$. See showLabels for other options.
id.cex size of text for point labels
id.col color for point labels
$x l a b \quad$ label for horizontal (theoretical quantiles) axis
ylab label for vertical (empirical quantiles) axis
main plot title
what the name of the object plotted; used in the construction of main when that is not specified.
ylim limits for vertical axis. If not specified, the range of the confidence envelope is used.

## Details

The calculation of the confidence envelope follows that used in the SAS program, http://www. datavis.ca/sasmac/cqplot.html which comes from Chambers et al. (1983), Section 6.8.

The essential formula is

$$
S E\left(z_{(i)}\right)=\frac{\hat{\delta}}{\left.g\left(q_{i}\right)\right) \times \sqrt{\operatorname{fracp}_{i}\left(1-p_{i}\right.} n}
$$

where $z_{(i)}$ is the i-th order value of $D^{2}, \hat{\delta}$ is an estimate of the slope of the reference line obtained from the corresponding quartiles and $g\left(q_{i}\right)$ is the density of the chi square distribution at the quantile $q_{i}$.
Note that this confidence envelope applies only to the $D^{2}$ computed using the classical estimates of location and scatter. The car: : qqPlot () function provides for simulated envelopes, but only for a univariate measure. Oldford (2016) provides a general theory and methods for QQ plots.

## Value

Returns invisibly the vector of squared Mahalanobis distances corresponding to the rows of $x$ or the residuals of the model.

## Author(s)

Michael Friendly

## References

J. Chambers, W. S. Cleveland, B. Kleiner, P. A. Tukey (1983). Graphical methods for data analysis, Wadsworth.
R. W. Oldford (2016), "Self calibrating quantile-quantile plots", The American Statistician, 70, 74-90.

## See Also

Mahalanobis for calculation of Mahalanobis squared distance;
qqplot; qqPlot can give a similar result for Mahalanobis squared distances of data or residuals; qqtest has many features for all types of QQ plots.

## Examples

```
cqplot(iris[, 1:4])
iris.mod <- lm(as.matrix(iris[,1:4]) ~ Species, data=iris)
cqplot(iris.mod, id.n=3)
# compare with car::qqPlot
car::qqPlot(Mahalanobis(iris[, 1:4]), dist="chisq", df=4)
# Adopted data
Adopted.mod <- lm(cbind(Age2IQ, Age4IQ, Age8IQ, Age13IQ) ~ AMED + BMIQ,
            data=Adopted)
cqplot(Adopted.mod, id.n=3)
cqplot(Adopted.mod, id.n=3, method="mve")
# Sake data
Sake.mod <- lm(cbind(taste, smell) ~ ., data=Sake)
cqplot(Sake.mod)
cqplot(Sake.mod, method="mve", id.n=2)
# SocialCog data -- one extreme outlier
data(SocialCog)
SC.mlm <- lm(cbind(MgeEmotions,ToM, ExtBias, PersBias) ~ Dx,
    data=SocialCog)
cqplot(SC.mlm, id.n=1)
# data frame example: stackloss data
data(stackloss)
cqplot(stackloss[, 1:3], id.n=4) # very strange
cqplot(stackloss[, 1:3], id.n=4, detrend=TRUE)
cqplot(stackloss[, 1:3], id.n=4, method="mve")
cqplot(stackloss[, 1:3], id.n=4, method="mcd")
```

cross3d Draw a 3D cross in an rgl scene

## Description

Draws a 3D cross or axis vectors in an rgl scene.

## Usage

cross3d(centre $=\operatorname{rep}(0,3)$, scale $=\operatorname{rep}(1,3), \ldots)$

## Arguments

centre A scalar or vector of length 3, giving the centre of the 3D cross
scale A scalar or vector of length 3, giving the lengths of the arms of the 3D cross
... Other arguments, passed on to segments3d

## Value

Used for its side-effect, but returns (invisibly) a 6 by 3 matrix containing the end-points of three axes, in pairs.

## Author(s)

Michael Friendly

See Also
segments3d

Diabetes Diabetes Dataset

## Description

Reaven and Miller (1979) examined the relationship among blood chemistry measures of glucose tolerance and insulin in 145 nonobese adults. They used the PRIM9 system at the Stanford Linear Accelerator Center to visualize the data in 3D, and discovered a peculiar pattern that looked like a large blob with two wings in different directions.
After further analysis, the subjects were classified as subclinical (chemical) diabetics, overt diabetics and normals. This study was influential in defining the stages of development of Type 2 diabetes. Overt diabetes is the most advanced stage, characterized by elevated fasting blood glucose concentration and classical symptoms. Preceding overt diabetes is the latent or chemical diabetic stage, with no symptoms of diabetes but demonstrable abnormality of oral or intravenous glucose tolerance.

## Usage

data("Diabetes")

## Format

A data frame with 145 observations on the following 6 variables.
relwt relative weight, expressed as the ratio of actual weight to expected weight, given the person's height, a numeric vector
glufast fasting plasma glucose level, a numeric vector
glutest test plasma glucose level, a measure of glucose intolerance, a numeric vector
instest plasma insulin during test, a measure of insulin response to oral glucose, a numeric vector sspg steady state plasma glucose, a measure of insulin resistance, a numeric vector group diagnostic group, a factor with levels Normal Chemical_Diabetic Overt_Diabetic

## Details

glutest was defined as the "area under the plasma glucose curve for the three hour oral glucose tolerance test." Reaven \& Miller refer to this variable as "Glucose area".
instest was defined as the "area under the plasma insulin curve", and is referred to in the paper as "Insulin area".
This study was influential in defining the stages of development of Type 2 diabetes. Overt diabetes is the most advanced stage, characterized by elevated fasting blood glucose concentration and classical symptoms. Preceding overt diabetes is the latent or chemical diabetic stage, with no symptoms of diabetes but demonstrable abnormality of oral or intravenous glucose tolerance.

## Source

Andrews, D. F. \& Herzberg, A. M. (1985). Data: A Collection of Problems from Many Fields for the Student and Research Worker, Springer-Verlag, Ch. 36.
Friendly, M. (1991). SAS System for Statistical Graphics, Cary, NC: SAS Institute.

## References

Reaven, G. M. and Miller, R. G. (1979). An attempt to define the nature of chemical diabetes using a multidimensional analysis. Diabetologia, 16, 17-24.

## Examples

```
data(Diabetes)
col <- c("blue", "red", "darkgreen")[Diabetes$group]
pch <- c(16,15,17)[Diabetes$group]
# a perplexing plot, similar to Fig 2, but with a loess smooth
plot(instest ~ glutest, data=Diabetes, pch=16,
cex.lab=1.25,
xlab="Glucose area (glutest)",
ylab="Insulin area (instest)")
lines( loess.smooth(Diabetes$glutest, Diabetes$instest), col="blue", lwd=2)
# scatterplot matrix, colored by group
plot(Diabetes[,1:5], col=col, pch=pch)
# covariance ellipses
covEllipses(Diabetes[,2:5], Diabetes$group, fill=TRUE, pooled=FALSE,
col=col)
covEllipses(Diabetes[,2:5], Diabetes$group, fill=TRUE, pooled=FALSE,
col=col, variables=1:4)
# Box's M test
```

```
diab.boxm <- boxM(Diabetes[,2:5], Diabetes$group)
diab.boxm
plot(diab.boxm)
# heplots
diab.mlm <- lm(cbind(glufast, glutest, instest, sspg) ~ group, data=Diabetes)
heplot(diab.mlm)
pairs(diab.mlm, fill=TRUE, fill.alpha=0.1)
```

```
ellipse3d.axes Draw axes of a 3D ellipsoid
```


## Description

A function to draw the major axes of a 3D ellipsoid from a correlation, covariance or sums of squares and cross products matrix.

## Usage

ellipse3d.axes(x, centre $=c(0,0,0)$, scale $=c(1,1,1)$,
level $=0.95, \mathrm{t}=\operatorname{sqrt}(\mathrm{qchisq}(\mathrm{level}, 3))$, which $=1: 3$,
labels = TRUE, label.ends=c $(2,4,6), \ldots)$

## Arguments

x
centre
scale
level The confidence level of a simultaneous confidence region. The default is 0.95 , for a $95 \%$ region. This is used to control the size of the ellipsoid.
t
which This parameter selects which variables from the object will be plotted. The default is the first 3 .
labels Either a logical value, a character string, or a character vector of length 3. If TRUE, the default, the axes are labeled PC1, PC2, PC3. If a single character string, the digits $1,2,3$ are pasted on the end.
label.ends A vector of length 3 indicating which ends of the axes should be labeled, corresponding to a selection of rows of the $6 \times 3$ matrix of axes end points. Default: $c(2,4,6)$.
$\ldots \quad$ Other arguments passed to segments3d and text3d.

## Value

Returns a $6 \times 3$ matrix containing the end points of the three axis lines in pairs by rows.

## Author(s)

Michael Friendly

## See Also

segments3d, text3d, ellipse3d

## Examples

```
data(iris)
iris3 <- iris[,1:3]
cov <- cov(iris3)
mu <- colMeans(iris3)
col <-c("blue", "green", "red")[iris$Species]
library(rgl)
plot3d(iris3, type="s", size=0.4, col=col, cex=2, box=FALSE, aspect="iso")
plot3d( ellipse3d(cov, centre=mu, level=0.68), col="gray", alpha=0.2, add = TRUE)
axes <- ellipse3d.axes(cov, centre=mu, level=0.68, color="gray", lwd=2)
```

etasq Measures of Partial Association (Eta-squared) for Linear Models

## Description

Calculates partial eta-squared for linear models or multivariate analogs of eta-squared (or $\mathrm{R}^{\wedge} 2$ ), indicating the partial association for each term in a multivariate linear model. There is a different analog for each of the four standard multivariate test statistics: Pillai's trace, Hotelling-Lawley trace, Wilks' Lambda and Roy's maximum root test.

## Usage

```
etasq(x, ...)
## S3 method for class 'lm'
etasq(x, anova = FALSE, partial = TRUE, ...)
## S3 method for class 'mlm'
etasq(x, ...)
## S3 method for class 'Anova.mlm'
etasq(x, anova = FALSE, ...)
```


## Arguments

X
anova A logical, indicating whether the result should also contain the test statistics produced by Anova().
partial A logical, indicating whether to calculate partial or classical eta^2.
Other arguments passed down to Anova.

## Details

For univariate linear models, classical $\eta^{2}=\mathrm{SSH} / \mathrm{SST}$ and partial $\eta^{2}=\mathrm{SSH} /(\mathrm{SSH}+\mathrm{SSE})$. These are identical in one-way designs.
Partial eta-squared describes the proportion of total variation attributable to a given factor, partialing out (excluding) other factors from the total nonerror variation. These are commonly used as measures of effect size or measures of (non-linear) strength of association in ANOVA models.
All multivariate tests are based on the $s=\min \left(p, d f_{h}\right)$ latent roots of $H E^{-1}$. The analogous multivariate partial $\eta^{2}$ measures are calculated as:

Pillai's trace (V) $\eta^{2}=V / s$
Hotelling-Lawley trace (T) $\eta^{2}=T /(T+s)$
Wilks' Lambda (L) $\eta^{2}=L^{1 / s}$
Roy's maximum root ( $\mathbf{R}$ ) $\eta^{2}=R /(R+1)$

## Value

When anova=FALSE, a one-column data frame containing the eta-squared values for each term in the model.

When anova=TRUE, a 5 -column ( lm ) or 7 -column ( mlm ) data frame containing the eta-squared values and the test statistics produced by print. Anova() for each term in the model.

## Author(s)

Michael Friendly

## References

Muller, K. E. and Peterson, B. L. (1984). Practical methods for computing power in testing the Multivariate General Linear Hypothesis Computational Statistics and Data Analysis, 2, 143-158.

Muller, K. E. and LaVange, L. M. and Ramey, S. L. and Ramey, C. T. (1992). Power Calculations for General Linear Multivariate Models Including Repeated Measures Applications. Journal of the American Statistical Association, 87, 1209-1226.

## See Also

Anova

## Examples

```
data(Soils) # from car package
soils.mod <- lm(cbind(pH,N,Dens,P,Ca,Mg,K,Na,Conduc) ~ Block + Contour*Depth, data=Soils)
#Anova(soils.mod)
etasq(Anova(soils.mod))
etasq(soils.mod) # same
etasq(Anova(soils.mod), anova=TRUE)
etasq(soils.mod, test="Wilks")
etasq(soils.mod, test="Hotelling")
```

FootHead Head measurements of football players

## Description

Data collected as part of a preliminary study examining the relation between football helmet design and neck injuries. There are 30 subjects in each of three groups: High school football players, college players and non-football players.

## Usage

data(FootHead)

## Format

A data frame with 90 observations on the following 7 variables.
group a factor with levels High school College Non-football
width a numeric vector: head width at widest dimension
circum a numeric vector: head circumference
front.back a numeric vector: front to back distance at eye level
eye.top a numeric vector: eye to top of head
ear.top a numeric vector:ear to top of head
jaw a numeric vector: jaw width

## Source

Rencher, A. C. (1995), Methods of Multivariate Analysis, New York: Wiley, Table 8.3.

## Examples

```
    data(FootHead)
    str(FootHead)
    require(car)
    # use Helmert contrasts for group
    contrasts(FootHead$group) <- contr.helmert
    contrasts(FootHead$group)
    foot.mod <- lm(cbind(width, circum,front.back,eye.top,ear.top,jaw)~group, data=FootHead)
    Manova(foot.mod)
    # show the HE plot for the first two variables
    heplot(foot.mod, main="HE plot for width and circumference", fill=TRUE,
    col=c("red", "blue"))
    # show it with tests of Helmert contrasts
    heplot(foot.mod, hypotheses=list("group.1"="group1","group.2"="group2"),
    col=c("red", "blue", "green3", "green3" ),
    main="HE plot with orthogonal Helmert contrasts")
    # show all pairwise HE plots
    pairs(foot.mod)
    # ... with tests of Helmert contrasts
    pairs(foot.mod, hypotheses=list("group.1"="group1","group.2"="group2"),
    col=c("red", "blue", "green3", "green3"), hyp.labels=FALSE)
    # see that the hypothesis for groups really is 2D
    if(requireNamespace("rgl")){
    heplot3d(foot.mod, variables=c(1,2,6),
    hypotheses=list("group.1"="group1", "group.2"="group2"),
    col=c("red", "blue", "green3", "green3"), wire=FALSE)
}
```

gsorth

Orthogonalize successive columns of a data frame or matrix

## Description

gsorth uses sequential, orthogonal projections, as in the Gram-Schmidt method, to transform a matrix or numeric columns of a data frame into an uncorrelated set, possibly retaining the same column means and standard deviations as the original.
In statistical applications, interpretation depends on the order of the variables orthogonalized. In multivariate linear models, orthogonalizing the response, Y variables provides the equivalent of step-down tests, where Y1 is tested alone, and then Y2.1, Y3.12, etc. can be tested to determine their additional contributions over the previous response variables.

Similarly, orthogonalizing the model X variables provides the equivalent of Type I tests, such as provided by anova.

## Usage

gsorth(y, order, recenter $=$ TRUE, rescale $=$ TRUE, adjnames $=$ TRUE)

## Arguments

$y \quad$ A numeric data frame or matrix
order An integer vector specifying the order of and/or a subset of the columns of $y$ to be orthogonalized. If missing, order=1:p where $p=n \operatorname{col}(y)$.
recenter If TRUE, the result has same column means as original; else means $=0$ for cols 2:p.
rescale If TRUE, the result has same column standard deviations as original; else sd = residual variance for cols $2: p$
adjnames If TRUE, the column names of the result are adjusted to the form Y1, Y2.1, Y3.12, by adding the suffixes '. 1 ', '. 12 ', etc. to the original column names.

## Details

The method is equivalent to setting each of columns 2:p to the residuals from a linear regression of that column on all prior columns, i.e.,

```
z[,j]<-resid( lm( z[,j] ~ as.matrix(z[,1:(j-1)]),data=z))
```

However, for accuracy and speed the transformation is carried out using the QR decomposition.

## Value

Returns a matrix or data frame with uncorrelated columns. Row and column names are copied to the result.

## Author(s)

Michael Friendly

## See Also

qr,

## Examples

```
GSiris <- gsorth(iris[,1:4])
GSiris <- gsorth(iris, order=1:4) # same, using order
str(GSiris)
zapsmall(cor(GSiris))
colMeans(GSiris)
# sd(GSiris) -- sd(<matrix>) now deprecated
apply(GSiris, 2, sd)
# orthogonalize Y side
GSiris <- data.frame(gsorth(iris[,1:4]), Species=iris$Species)
iris.mod1 <- lm(as.matrix(GSiris[,1:4]) ~ Species, data=GSiris)
```

```
Anova(iris.mod1)
# orthogonalize X side
rohwer.mod <- lm(cbind(SAT, PPVT, Raven) ~ n + s + ns + na + ss, data=Rohwer)
Anova(rohwer.mod)
# type I tests for Rohwer data
Rohwer.orth <- cbind(Rohwer[,1:5], gsorth(Rohwer[, c("n", "s", "ns", "na", "ss")], adjnames=FALSE))
rohwer.mod1 <- lm(cbind(SAT, PPVT, Raven) ~ n + s + ns + na + ss, data=Rohwer.orth)
Anova(rohwer.mod1)
# compare with anova()
anova(rohwer.mod1)
# compare heplots for original Xs and orthogonalized, Type I
heplot(rohwer.mod)
heplot(rohwer.mod1)
```

Headache Treatment of Headache Sufferers for Sensitivity to Noise

## Description

A study was conducted investigating the effectiveness of different kinds of psychological treatment on the sensitivity of headache sufferers to noise, described in Hand and Taylor (1987), Study E.
In a pre-post design, 98 patients were first assessed for the volume of noise which they found uncomfortable (U) and definitely uncomfortable (DU). They were then given relaxation training, where they listened to the noise at the DU level and given instruction breathing techniques and the use of visual imagery to distract them from discomfort. One of four treatments was then applied, and all patients were reassessed for the noise volume they considered uncomfortable ( U ) and definitely uncomfortable (DU).

## Usage

data(Headache)

## Format

A data frame with 98 observations on the following 6 variables.
type Type of headache, a factor with levels Migrane Tension treatment Treatment group, a factor with levels T1 T2 T3 Control. See Details
u1 Noise level rated as Uncomfortable, initial measure
du1 Noise level rated as Definitely Uncomfortable, initial measure
u2 Noise level rated as Uncomfortable, final measure
du2 Noise level rated as Definitely Uncomfortable, final measure

## Details

The treatments are described as follows:
T1 Listened again to the tone at their initial DU level, for the same amount of time they were able to tolerate it before.
T2 Same as T1, with one additional minute exposure
T3 Same as T2, but were explicitly instructed to use the relaxation techniques
Control These subject experienced no further exposure to the noise tone until the final sensitivity measures were taken

Hand and Taylor described several substantive hypotheses related to the differences among treatments. In the Headache data frame, these have been included as contrasts (Headache\$treatment)

## Source

D. J. Hand and C. C. Taylor (1987). Multivariate analysis of variance and repeated measures: a practical approach for behavioural scientists London: Chapman and Hall. ISBN: 0412258005. Table E.1.

## Examples

```
data(Headache)
str(Headache)
# basic MLM, specifying between-S effects
headache.mod <- lm(cbind(u1, du1, u2, du2) ~ type * treatment, data=Headache)
################################
## between-S tests
################################
Anova(headache.mod, test="Roy")
# test each contrast separately
print(linearHypothesis(headache.mod, hypothesis="treatment1", test="Roy"), SSP=FALSE)
print(linearHypothesis(headache.mod, hypothesis="treatment2", test="Roy"), SSP=FALSE)
print(linearHypothesis(headache.mod, hypothesis="treatment3", test="Roy"), SSP=FALSE)
heplot(headache.mod, variables=c(1,3),
hypotheses=paste("treatment", 1:3, sep=""),
hyp.labels=c("extra.exp", "no.inst", "explicit.inst"),
xlab="u1: Initial sensitivity", ylab="u2: Final sensitivity",
main="Headache data: Unpleasant levels")
abline(0, 1, lty=5, col="gray")
heplot(headache.mod, variables=c(2,4),
hypotheses=paste("treatment", 1:3, sep=""),
xlab="du1: Initial sensitivity", ylab="du2: Final sensitivity",
main="Headache data: Definitely Unpleasant levels")
abline(0, 1, lty=5, col="gray")
```

```
pairs(headache.mod)
##############################
# between-S and within-S tests
###############################
idata = expand.grid(level=factor(c("U", "DU")), phase=factor(1:2))
Anova(headache.mod, idata=idata, idesign=~level*phase)
```

heplot Two-Dimensional HE Plots

## Description

This function plots ellipses representing the hypothesis and error sums-of-squares-and-products matrices for terms and linear hypotheses in a multivariate linear model. These include MANOVA models (all explanatory variables are factors), multivariate regression (all quantitative predictors), MANCOVA models, homogeneity of regression, as well as repeated measures designs treated from a multivariate perspective.

## Usage

```
heplot(mod, ...)
## S3 method for class 'mlm'
heplot(mod, terms, hypotheses, term.labels = TRUE,
    hyp.labels = TRUE, err.label="Error", label.pos=NULL,
    variables = 1:2, error.ellipse = !add,
    factor.means = !add, grand.mean = !add, remove.intercept = TRUE,
    type = c("II", "III", "2", "3"), idata=NULL, idesign=NULL,
icontrasts=c("contr.sum", "contr.poly"),
imatrix=NULL, iterm=NULL, markH0=!is.null(iterm),
    manova, size = c("evidence", "effect.size"),
    level = 0.68, alpha = 0.05, segments = 60,
    center.pch = "+", center.cex=2,
    col = getOption("heplot.colors",
            c("red", "blue", "black", "darkgreen",
                            "darkcyan","magenta", "brown","darkgray")),
    lty = 2:1, lwd = 1:2,
    fill=FALSE, fill.alpha=0.3,
    xlab, ylab, main = "", xlim, ylim, axes=TRUE, offset.axes,
    add = FALSE, verbose = FALSE, warn.rank = FALSE, ...)
```


## Arguments

mod a model object of class "mlm".

| terms | a logical value or character vector of terms in the model for which to plot hy- <br> pothesis matrices; if missing or TRUE, defaults to all terms; if FALSE, no terms <br> are plotted. |
| :--- | :--- |
| hypotheses | optional list of linear hypotheses for which to plot hypothesis matrices; hypothe- <br> ses are specified as for the linearHypothesis function in the car package; the <br> list elements can be named, in which case the names are used. |
| term.labels | logical value or character vector of names for the terms to be plotted. If TRUE (the <br> default) the names of the terms are used; if FALSE, term labels are not plotted. |
| hyp.labels | logical value or character vector of names for the hypotheses to be plotted. If |
| TRUE (the default) the names of components of the list of hypotheses are used; |  |
| if FALSE, hypothesis labels are not plotted. |  |


| iterm | For repeated measures designs, you must specify one intra-subject term (a char- <br> acter string) to select the SSPE (E) matrix used in the HE plot. Hypothesis terms <br> plotted include the iterm effect as well as all interactions of iterm with terms. |
| :--- | :--- |
| markH0 | A logical value (or else a list of arguments to mark. H0) used to draw cross-hairs <br> and a point indicating the value of a point null hypothesis. The default is TRUE <br> if iterm is non-NULL. <br> optional Anova.mlm object for the model; if absent a MANOVA is computed. |
| manova | Specifying the argument can therefore save computation in repeated calls. |
| size | how to scale the hypothesis ellipse relative to the error ellipse; if "evidence", <br> the default, the scaling is done so that a "significant" hypothesis ellipse at level <br> alpha extends outside of the error ellipse; if "effect.size", the hypothesis |
| ellipse is on the same scale as the error ellipse. |  |
| equivalent coverage of ellipse for normally-distributed errors, defaults to 0.68, |  |
| giving a standard 1 SD bivariate ellipse. |  |


| ylab <br> main <br> xlim <br> ylim <br> axes <br> offset. axes | y-axis label; defaults to name of the y variable. <br> main plot label; defaults to "". |
| :--- | :--- |
| x-axis limits; if absent, will be computed from the data. |  |
| y-axis limits; if absent, will be computed from the data. |  |
| add | Whether to draw the $x, y$ axes; defaults to TRUE <br> proportion to extend the axes in each direction if computed from the data; op- <br> tional. <br> if TRUE, add to the current plot; the default is FALSE. If TRUE, the error ellipse is <br> not plotted. <br> if TRUE, print the MANOVA table and details of hypothesis tests; the default is |
| verbose | FALSE. |
| warn.rank | if TRUE, do not suppress warnings about the rank of the hypothesis matrix when <br> the ellipse collapses to a line; the default is FALSE. |
| $\ldots$ | arguments to pass down to plot, text, and points. |

## Details

The heplot function plots a representation of the covariance ellipses for hypothesized model terms and linear hypotheses $(\mathrm{H})$ and the corresponding error $(\mathrm{E})$ matrices for two response variables in a multivariate linear model (mlm).

The plot helps to visualize the nature and dimensionality response variation on the two variables jointly in relation to error variation that is summarized in the various multivariate test statistics (Wilks' Lambda, Pillai trace, Hotelling-Lawley trace, Roy maximum root). Roy's maximum root test has a particularly simple visual interpretation, exploited in the size="evidence" version of the plot. See the description of argument alpha.
For a 1 df hypothesis term (a quantitative regressor, a single contrast or parameter test), the H matrix has rank 1 (one non-zero latent root of $H E^{-1}$ ) and the H "ellipse" collapses to a degenerate line.
Typically, you fit a mlm with mymlm <-lm(cbind ( $\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3, \ldots$ ) $\sim$ modelterms), and plot some or all of the modelterms with heplot(mymlm,...). Arbitrary linear hypotheses related to the terms in the model (e.g., contrasts of an effect) can be included in the plot using the hypotheses argument. See linearHypothesis for details.
For repeated measure designs, where the response variables correspond to one or more variates observed under a within-subject design, between-subject effects and within-subject effects must be plotted separately, because the error terms (E matrices) differ. When you specify an intra-subject term (iterm), the analysis and HE plots amount to analysis of the matrix $\mathbf{Y}$ of responses postmultiplied by a matrix $\mathbf{M}$ determined by the intra-subject design for that term. See Friendly (2010) or the vignette ("repeated") in this package for an extended discussion and examples.
The related candisc package provides functions for visualizing a multivariate linear model in a low-dimensional view via a generalized canonical discriminant analyses. heplot.candisc and heplot3d.candisc provide a low-rank 2D (or 3D) view of the effects for a given term in the space of maximum discrimination.

When an element of fill is TRUE, the ellipse outline is drawn using the corresponding color in col , and the interior is filled with a transparent version of this color specified in fill.alpha. To produce filled (non-degenerate) ellipses without the bounding outline, use a value of lty=0 in the corresponding position.

## Value

The function invisibly returns an object of class "heplot", with coordinates for the various hypothesis ellipses and the error ellipse, and the limits of the horizontal and vertical axes. These may be useful for adding additional annotations to the plot, using standard plotting functions. (No methods for manipulating these objects are currently available.)

The components are:

H a list containing the coordinates of each ellipse for the hypothesis terms
E a matrix containing the coordinates for the error ellipse
center $\quad \mathrm{x}, \mathrm{y}$ coordinates of the centroid
xlim $\quad x$-axis limits
ylim $\quad y$-axis limits
radius the radius for the unit circles used to generate the ellipses

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples Journal of Statistical Software, 17(6), 1-42.
doi: 10.18637/jss.v017.i06
Friendly, M. (2007). HE plots for Multivariate General Linear Models. Journal of Computational and Graphical Statistics, 16(2) 421-444. http://datavis.ca/papers/jcgs-heplots.pdf

Friendly, Michael (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04
Fox, J., Friendly, M. \& Weisberg, S. (2013). Hypothesis Tests for Multivariate Linear Models Using the car Package. The R Journal, 5(1), https://journal.r-project.org/archive/2013-1/ fox-friendly-weisberg.pdf.

Friendly, M. \& Sigal, M. (2014) Recent Advances in Visualizing Multivariate Linear Models. Revista Colombiana de Estadistica, 37, 261-283.

## See Also

Anova, linearHypothesis for details on testing MLMs.
heplot1d, heplot3d, pairs.mlm, mark. H0 for other HE plot functions. coefplot.mlm for plotting confidence ellipses for parameters in MLMs.
trans. colors for calculation of transparent colors. label. ellipse for labeling positions in plotting H and E ellipses.
candisc, heplot. candisc for reduced-rank views of mlms in canonical space.

## Examples

```
## iris data
contrasts(iris$Species)<-matrix(c(0,-1,1, 2, -1, -1), 3,2)
contrasts(iris$Species)
iris.mod <- lm(cbind(Sepal.Length, Sepal.Width, Petal.Length, Petal.Width) ~
Species, data=iris)
hyp <- list("V:V"="Species1","S:VV"="Species2")
heplot(iris.mod, hypotheses=hyp)
# compare with effect-size scaling
heplot(iris.mod, hypotheses=hyp, size="effect", add=TRUE)
# try filled ellipses
heplot(iris.mod, hypotheses=hyp, fill=TRUE, fill.alpha=0.2, col=c("red", "blue"))
heplot(iris.mod, hypotheses=hyp, fill=TRUE, col=c("red", "blue"), lty=c(0,0,1,1))
# vary label position and fill.alpha
heplot(iris.mod, hypotheses=hyp, fill=TRUE, fill.alpha=c(0.3,0.1), col=c("red", "blue"),
        lty=c(0,0,1,1), label.pos=0:3)
hep <-heplot(iris.mod, variables=c(1,3), hypotheses=hyp)
str(hep)
# all pairs
pairs(iris.mod, hypotheses=hyp, hyp.labels=FALSE)
## Pottery data, from car package
data(Pottery)
pottery.mod <- lm(cbind(Al, Fe, Mg, Ca, Na) ~ Site, data=Pottery)
heplot(pottery.mod)
heplot(pottery.mod, terms=FALSE, add=TRUE, col="blue",
    hypotheses=list(c("SiteCaldicot = 0", "SiteIsleThorns=0")),
    hyp.labels="Sites Caldicot and Isle Thorns")
## Rohwer data, multivariate multiple regression/ANCOVA
#-- ANCOVA, assuming equal slopes
rohwer.mod <- lm(cbind(SAT, PPVT, Raven) ~ SES + n + s + ns + na + ss, data=Rohwer)
Anova(rohwer.mod)
col <- c("red", "black", "blue", "cyan", "magenta", "brown", "gray")
heplot(rohwer.mod, col=col)
# Add ellipse to test all 5 regressors
heplot(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")), col=col, fill=TRUE)
# View all pairs
pairs(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")))
# or 3D plot
if(requireNamespace("rgl")){
col <- c("pink", "black", "blue", "cyan", "magenta", "brown", "gray")
heplot3d(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")), col=col)
}
```


## Description

This function plots a 1-dimensional representation of the hypothesis (H) and error (E) sums-of-squares-and-products matrices for terms and linear hypotheses in a multivariate linear model.
In particular, for a given response, the 1-D representations of H and E matrices correspond to line segments. The E "ellipse" is shown as a filled rectangle whose width equals the mean squared error for that response. The H "ellipse" for each model term is shown as a line segment whose length represents either the size of the effect or the evidence for that effect.

## This version is an initial sketch. Details of the implementation are subject to change.

```
Usage
    heplot1d(mod, ...)
    ## S3 method for class 'mlm'
    heplot1d(mod, terms, hypotheses,
        term.labels = TRUE, hyp.labels = TRUE,
    variables = 1, error.ellipse = !add, factor.means = !add, grand.mean = !add,
    remove.intercept = TRUE, type = c("II", "III", "2", "3"),
    idata=NULL, idesign=NULL, icontrasts=c("contr.sum", "contr.poly"),
    imatrix=NULL, iterm=NULL, manova,
    size = c("evidence", "effect.size"), level = 0.68, alpha = 0.05,
    center.pch = "|",
        col = getOption("heplot.colors",
            c("red", "blue", "black", "darkgreen",
                        "darkcyan","magenta", "brown","darkgray")),
    lty = 2:1, lwd = 1:2, xlab, main = "",
    xlim, axes = TRUE, offset.axes, add = FALSE, verbose = FALSE, ...)
```


## Arguments

\(\left.\begin{array}{ll}mod \& a model object of class "mlm". <br>
a logical value or character vector of terms in the model for which to plot hy- <br>
pothesis matrices; if missing or TRUE, defaults to all terms; if FALSE, no terms <br>

are plotted.\end{array}\right\}\)| optional list of linear hypotheses for which to plot hypothesis matrices; hypothe- |
| :--- |
| ses are specified as for the linearHypothesis function in the car package; the |
| list elements can be named, in which case the names are used. |


| error.ellipse | if TRUE, plot the error ellipse; defaults to TRUE, if the argument add is FALSE (see below). |
| :---: | :---: |
| factor.means | logical value or character vector of names of factors for which the means are to be plotted, or TRUE or FALSE; defaults to TRUE, if the argument add is FALSE (see below). |
| grand.mean | if TRUE, plot the centroid for all of the data; defaults to TRUE, if the argument add is FALSE (see below). |
| remove.intercept |  |
|  | if TRUE (the default), do not plot the ellipse for the intercept even if it is in the MANOVA table. |
| type | "type" of sum-of-squares-and-products matrices to compute; one of "II", "III", " 2 ", or " 3 ", where "II" is the default (and " 2 " is a synonym). |
| idata | an optional data frame giving a factor or factors defining the intra-subject model for multivariate repeated-measures data. See Details of Anova for an explanation of the intra-subject design and for further explanation of the other arguments relating to intra-subject factors. |
| idesign | a one-sided model formula using the "data" in idata and specifying the intrasubject design for repeated measure models. |
| icontrasts | names of contrast-generating functions to be applied by default to factors and ordered factors, respectively, in the within-subject "data"; the contrasts must produce an intra-subject model matrix in which different terms are orthogonal. The default is c("contr.sum", "contr.poly"). |
| imatrix | In lieu of idata and idesign, you can specify the intra-subject design matrix directly via imatrix, in the form of list of named elements. Each element gives the columns of the within-subject model matrix for an intra-subject term to be tested, and must have as many rows as there are responses; the columns of the within-subject model matrix for different terms must be mutually orthogonal. This functionality requires car version 2.0 or later. |
| iterm | For repeated measures designs, you must specify one intra-subject term (a character string) to select the SSPE (E) matrix used in the HE plot. Hypothesis terms plotted include the iterm effect as well as all interactions of iterm with terms. |
| manova | optional Anova.mlm object for the model; if absent a MANOVA is computed. Specifying the argument can therefore save computation in repeated calls. |
| size | how to scale the hypothesis ellipse relative to the error ellipse; if "evidence", the default, the scaling is done so that a "significant" hypothesis ellipse extends outside of the error ellipse; if "effect.size", the hypothesis ellipse is on the same scale as the error ellipse. |
| level | equivalent coverage of ellipse for normally-distributed errors, defaults to 0.68. |
| alpha | significance level for Roy's greatest-root test statistic; if size="evidence", then the hypothesis ellipse is scaled so that it just touches the error ellipse at the specified alpha level; a larger hypothesis ellipse therefore indicates statistical significance; defaults to 0.05 . |
| center.pch | character to use in plotting the centroid of the data; defaults to " $\mid$ ". |


| col | a color or vector of colors to use in plotting ellipses; the first color is used for the error ellipse; the remaining colors - recycled as necessary - are used for the hypothesis ellipses. A single color can be given, in which case it is used for all ellipses. For convenience, the default colors for all heplots produced in a given session can be changed by assigning a color vector via options (heplot. colors =c (. . .). Otherwise, the default colors are c("red", "blue", "black", "darkgreen", "darkcyan" , "mas |
| :---: | :---: |
| lty | vector of line types to use for plotting the ellipses; the first is used for the error ellipse, the rest - possibly recycled - for the hypothesis ellipses; a single line type can be given. Defaults to $2: 1$. |
| lwd | vector of line widths to use for plotting the ellipses; the first is used for the error ellipse, the rest - possibly recycled - for the hypothesis ellipses; a single line width can be given. Defaults to 1:2. |
| xlab | x -axis label; defaults to name of the x variable. |
| main | main plot label; defaults to "". |
| $x \mathrm{lim}$ | x -axis limits; if absent, will be computed from the data. |
| axes | Whether to draw the x, y axes; defaults to TRUE |
| offset.axes | proportion to extend the axes in each direction if computed from the data; optional. |
| add | if TRUE, add to the current plot; the default is FALSE. If TRUE, the error ellipse is not plotted. |
| verbose | if TRUE, print the MANOVA table and details of hypothesis tests; the default is FALSE. |
|  | arguments to pass down to plot, text, and points. |

## Value

The function invisibly returns an object of class "heplot1d", with coordinates for the various hypothesis ellipses and the error ellipse, and the limits of the horizontal and vertical axes. (No methods for manipulating these objects are currently available.)
The components are:

| H | ranges for the hypothesis terms |
| :--- | :--- |
| E | range for E |
| xlim | x-axis limits |

## Author(s)

Michael Friendly

## See Also

Anova, linearHypothesis for hypothesis tests in mlms
heplot, heplot3d, pairs.mlm for other HE plot methods

## Examples

```
## Plastics data
plastic.mod <- lm(cbind(tear, gloss, opacity) ~ rate*additive, data=Plastic)
heplot1d(plastic.mod, col=c("pink","blue"))
heplot1d(plastic.mod, col=c("pink","blue"),variables=2)
heplot1d(plastic.mod, col=c("pink","blue"), variables=3)
## Bees data
bees.mod <- lm(cbind(Iz,Iy) ~ caste*treat*time, data=Bees)
heplot1d(bees.mod)
heplot1d(bees.mod, variables=2)
```

heplot3d Three-Dimensional HE Plots

## Description

This function plots ellipsoids in 3D representing the hypothesis and error sums-of-squares-andproducts matrices for terms and linear hypotheses in a multivariate linear model.

## Usage

```
heplot3d(mod, ...)
```

\#\# S3 method for class 'mlm'
heplot3d(mod, terms, hypotheses, term.labels = TRUE,
hyp.labels = TRUE, err.label="Error", variables = 1:3, error.ellipsoid = !add,
factor.means = !add, grand.mean = !add, remove.intercept = TRUE,
type = c("II", "III", "2", "3"), idata=NULL, idesign=NULL,
icontrasts=c("contr.sum", "contr.poly"), imatrix=NULL, iterm=NULL,
manova, size = c("evidence", "effect.size"),
level $=0.68$, alpha $=0.05$, segments $=40$,
col=getOption("heplot3d.colors",
c("red", "blue", "black", "darkgreen",
"darkcyan", "magenta", "brown","darkgray")),
$1 w d=c(1,4)$,
shade=TRUE, shade.alpha=0.2, wire=c(TRUE,FALSE),
bg.col = c("white", "black"),
fogtype = c("none", "exp2", "linear", "exp"),
fov $=30$, offset $=0.01$,
xlab, ylab, zlab, xlim, ylim, zlim,
add $=$ FALSE, verbose $=$ FALSE,
warn.rank = FALSE, ...)

## Arguments

$\left.\begin{array}{ll}\text { mod } & \text { a model object of class "mlm". } \\ \text { a logical value or character vector of terms in the model for which to plot hy- } \\ \text { terms } \\ \text { pothesis matrices; if missing or TRUE, defaults to all terms; if FALSE, no terms } \\ \text { are plotted. } \\ \text { optional list of linear hypotheses for which to plot hypothesis matrices; hypothe- } \\ \text { ses are specified as for the linearHypothesis function in the car package; the } \\ \text { list elements can be named, in which case the names are used. }\end{array}\right\}$
\(\left.$$
\begin{array}{ll}\text { iterm } & \begin{array}{l}\text { For repeated measures designs, you must specify one intra-subject term (a char- } \\
\text { acter string) to select the SSPE (E) matrix used in the HE plot. Hypothesis terms } \\
\text { plotted include the iterm effect as well as all interactions of iterm with terms. }\end{array} \\
\text { manova } & \begin{array}{l}\text { optional Anova.mlm object for the model; if absent a MANOVA is computed. } \\
\text { Specifying the argument can therefore save computation in repeated calls. }\end{array} \\
\text { size } & \begin{array}{l}\text { how to scale the hypothesis ellipsoid relative to the error ellipsoid; if "evidence", } \\
\text { the default, the scaling is done so that a "significant" hypothesis ellipsoid ex- } \\
\text { tends outside of the error ellipsoid; if "effect.size", the hypothesis ellipsoid } \\
\text { is on the same scale as the error ellipsoid. }\end{array} \\
\text { level } & \begin{array}{l}\text { equivalent coverage of ellipsoid for normally-distributed errors, defaults to } 0.68 .\end{array}
$$ <br>
significance level for Roy's greatest-root test statistic; if size="evidence", then <br>

the hypothesis ellipsoid is scaled so that it just touches the error ellipsoid at the\end{array}\right]\)| specified alpha level; a larger hypothesis ellipsoid therefore indicates statistical |
| :--- |
| significance; defaults to 0.05. |


| zlim | z-axis limits; if absent, will be computed from the data. |
| :--- | :--- |
| add | if TRUE, add to the current plot; the default is FALSE. If TRUE, the error ellipsoid <br> is neither plotted nor returned in the output object. |
| verbose | if TRUE, print the MANOVA table and details of hypothesis tests; the default is <br> FALSE. |
| warn.rank | if TRUE, do not suppress warnings about the rank of the hypothesis matrix when <br> the ellipsoid collapses to an ellipse or line; the default is FALSE. |
| $\ldots$ | arguments passed from generic. |

## Details

When the H matrix for a term has rank $<3$, the ellipsoid collapses to an ellipse $(\operatorname{rank}(\mathrm{H})=2)$ or a line $(\operatorname{rank}(H)=1)$.
Rotating the plot can be particularly revealing, showing views in which H variation is particularly large or small in relation to E variation. See play3d and movie3d for details on creating animations.

The arguments xlim, ylim, and zlim can be used to expand the bounding box of the axes, but cannot decrease it.

## Value

heplot3d invisibly returns a list containing the bounding boxes of the error (E) ellipsoid and for each term or linear hypothesis specified in the call. Each of these is a $2 \times 3$ matrix with rownames "min" and "max" and colnames corresponding to the variables plotted. An additional component, center, contains the coordinates of the centroid in the plot.

The function also leaves an object named .frame in the global environment, containing the rgl object IDs for the axes, axis labels, and bounding box; these are deleted and the axes, etc. redrawn if the plot is added to.

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples Journal of Statistical Software, 17(6), 1-42. https://www. jstatsoft.org/v17/i06/

Friendly, M. (2007). HE plots for Multivariate General Linear Models. Journal of Computational and Graphical Statistics, 16(2) 421-444. http://datavis.ca/papers/jcgs-heplots.pdf

## See Also

Anova, linearHypothesis, for details on MANOVA tests and linear hypotheses
heplot, pairs.mlm, for other plotting methods for mlm objects
rgl-package, for details about 3D plots with rgl
heplot3d. candisc for 3D HE plots in canonical space.

## Examples

```
    # Soils data, from car package
    soils.mod <- lm(cbind(pH,N,Dens,P,Ca,Mg,K,Na,Conduc) ~ Block + Contour*Depth, data=Soils)
    Anova(soils.mod)
    heplot(soils.mod, variables=c("Ca", "Mg"))
    pairs(soils.mod, terms="Depth", variables=c("pH", "N", "P", "Ca", "Mg"))
    heplot3d(soils.mod, variables=c("Mg", "Ca", "Na"), wire=FALSE)
    # Plastic data
    plastic.mod <- lm(cbind(tear, gloss, opacity) ~ rate*additive, data=Plastic)
    ## Not run:
    heplot3d(plastic.mod, col=c("red", "blue", "brown", "green3"), wire=FALSE)
    ## End(Not run)
```

    Hernior Recovery from Elective Herniorrhaphy
    
## Description

A data set on measures of post-operative recovery of 32 patients undergoing an elective herniorrhaphy operation, in relation to pre-operative measures.

## Usage

data(Hernior)

## Format

A data frame with 32 observations on the following 9 variables.
age patient age
sex patient sex, a factor with levels f m
pstat physical status (ignoring that associated with the operation). A 1-5 scale, with $1=$ perfect health, $5=$ very poor health.
build body build, a 1-5 scale, with $1=$ emaciated, $2=$ thin, $3=$ average, $4=$ fat, $5=$ obese .
cardiac preoperative complications with heart, 1-4 scale, with $1=$ none, $2=$ mild, $3=$ moderate, $4=$ severe.
resp preoperative complications with respiration, 1-4 scale, with $1=$ none, $2=$ mild, $3=$ moderate, $4=$ severe.
leave condition upon leaving the recovery room, a 1-4 scale, with $1=$ routine recovery, $2=$ intensive care for observation overnight, $3=$ intensive care, with moderate care required, $4=$ intensive care, with moderate care required.
los length of stay in hospital after operation (days)
nurse level of nursing required one week after operation, a 1-5 scale, with $1=$ intense, $2=$ heavy, $3=$ moderate, $4=$ light, $5=$ none (?); see Details

## Details

leave, nurse and los are outcome measures; the remaining variables are potential predictors of recovery status.
The variable nurse is recorded as $1-4$, with remaining (20) entries entered as "-" in both sources. It is not clear whether this means "none" or NA. The former interpretation was used in constructing the R data frame, so nurse==5 for these observations. Using Hernior\$nurse[Hernior\$nurse==5] <-NA would change to the other interpretation, but render nurse useless in a multivariate analysis.
The ordinal predictors could instead be treated as factors, and there are also potential interactions to be explored.

## Source

Mosteller, F. and Tukey, J. W. (1977), Data analysis and regression, Reading, MA: Addison-Wesley. Data Exhibit 8, 567-568. Their source: A study by B. McPeek and J. P. Gilbert of the Harvard Anesthesia Center.

## References

Hand, D. J., Daly, F., Lunn, A. D., McConway, K. J. and Ostrowski, E. (1994), A Handbook of Small Data Sets, Number 484, 390-391.

## Examples

```
str(Hernior)
Hern.mod <- lm(cbind(leave, nurse, los) ~
                age + sex + pstat + build + cardiac + resp, data=Hernior)
Anova(Hern.mod, test="Roy") # actually, all tests are identical
# test overall regression
linearHypothesis(Hern.mod, c("age", "sexm", "pstat", "build", "cardiac", "resp"))
# joint test of age, sex & caridac
linearHypothesis(Hern.mod, c("age", "sexm", "cardiac"))
clr <- c("red", "darkgray", "blue", "darkgreen", "magenta", "brown", "black")
heplot(Hern.mod, col=clr)
pairs(Hern.mod, col=clr)
## Enhancing the pairs plot ...
# create better variable labels
vlab <- c("LeaveCondition\n(leave)", "NursingCare\n(nurse)", "LengthOfStay\n(los)")
# Add ellipse to test all 5 regressors simultaneously
hyp <- list("Regr" = c("age", "sexm", "pstat", "build", "cardiac", "resp"))
pairs(Hern.mod, hypotheses=hyp, col=clr, var.labels=vlab)
## Views in canonical space for the various predictors
if (require(candisc)) {
Hern.canL <- candiscList(Hern.mod)
plot(Hern.canL, term="age")
plot(Hern.canL, term="sex")
plot(Hern.canL, term="pstat") # physical status
}
```


## Description

Plot an interpolation between two related data sets, typically transformations of each other. This function is designed to be used in animations.
Points are plotted via the linear interpolation,

$$
X Y=X Y 1+\alpha(X Y 2-X Y 1)
$$

The function allows plotting of the data ellipse, the linear regression line, and line segments showing the movement of points.

## Usage

interpPlot(xy1, xy2, alpha, xlim, ylim, points=TRUE, add=FALSE, col=palette()[1], ellipse = FALSE, ellipse.args = NULL, abline=FALSE, col.lines = palette()[2], lwd=2, id.method = "mahal", labels=rownames(xy1), id.n = 0, id.cex = 1, id.col = palette()[1], segments=FALSE, segment.col="darkgray", ...)

## Arguments

| xy1 | First data set, a 2-column matrix or data.frame |
| :---: | :---: |
| xy2 | Second data set, a 2-column matrix or data.frame |
| alpha | The value of the interpolation fraction, typically (but not necessarily) $0<=$ alpha <=1). |
| xlim, ylim points | $x, y$ limits for the plot. If not specified, the function uses the ranges of rbind ( $x y 1, x y 2$ ). Logical. Whether to plot the points in the current interpolation? |
| col | Color for plotted points. |
| add | Logical. Whether to add to an existing plot? |
| ellipse | logical. TRUE to plot a dataEllipse |
| ellipse.args | other arguments passed to dataEllipse |
| abline | logical. TRUE to plot the linear regression line for XY |
| col.lines | line color |
| lwd | line width |
| id.method | How points are to be identified. See showLabels. |


| labels | observation labels |
| :--- | :--- |
| id.n | Number of points to be identified. If set to zero, no points are identified. |
| id.cex | Controls the size of the plotted labels. The default is 1 |
| id.col | Controls the color of the plotted labels. |
| segments | logical. TRUE to draw lines segments from xy1 to xy |
| segment.col | line color for segments |
| $\ldots$ | other arguments passed to plot() |

## Details

Interpolations other than linear can be obtained by using a non-linear series of alpha values. For example alpha=sin(seq $(0,1, .1) / \sin (1)$ will give a sinusoid interpolation.

## Value

Returns invisibly the interpolated XY points.

## Note

The examples here just use on-screen animations to the console graphics window. The animation package provides facilities to save these in various formats.

## Author(s)

Michael Friendly

## See Also

dataEllipse, showLabels, animation

## Examples

```
#################################################
# animate an AV plot from marginal to conditional
###################################################
data(Duncan, package="car")
duncmod <- lm(prestige ~ income + education, data=Duncan)
mod.mat <- model.matrix(duncmod)
# function to do an animation for one variable
dunc.anim <- function(variable, other, alpha=seq(0, 1, .1)) {
    var <- which(variable==colnames(mod.mat))
    duncdev <- scale(Duncan[,c(variable, "prestige")], scale=FALSE)
    duncav <- lsfit(mod.mat[, -var], cbind(mod.mat[, var], Duncan$prestige),
                intercept = FALSE)$residuals
    colnames(duncav) <- c(variable, "prestige")
    lims <- apply(rbind(duncdev, duncav),2,range)
```

```
    for (alp in alpha) {
        main <- if(alp==0) paste("Marginal plot:", variable)
            else paste(round(100*alp), "% Added-variable plot:", variable)
        interpPlot(duncdev, duncav, alp, xlim=lims[,1], ylim=lims[,2], pch=16,
            main = main,
            xlab = paste(variable, "| ", alp, other),
            ylab = paste("prestige | ", alp, other),
            ellipse=TRUE, ellipse.args=(list(levels=0.68, fill=TRUE, fill.alpha=alp/2)),
            abline=TRUE, id.n=3, id.cex=1.2, cex.lab=1.25)
        Sys.sleep(1)
    }
}
# show these in the R console
if(interactive()) {
dunc.anim("income", "education")
dunc.anim("education", "income")
}
#############################################
# correlated bivariate data with 2 outliers
# show rotation from data space to PCA space
#################################################
set.seed(123345)
x <- c(rnorm(100), 2, -2)
y <- c(x[1:100] + rnorm(100), -2, 2)
XY <- cbind(x=x, y=y)
rownames(XY) <- seq_along(x)
XY <- scale(XY, center=TRUE, scale=FALSE)
# start, end plots
dataEllipse(XY, pch=16, levels=0.68, id.n=2)
mod <- lm(y~x, data=as.data.frame(XY))
abline(mod, col="red", lwd=2)
pca <- princomp(XY, cor=TRUE)
scores <- pca$scores
dataEllipse(scores, pch=16, levels=0.68, id.n=2)
abline(lm(Comp.2 ~ Comp.1, data=as.data.frame(scores)), lwd=2, col="red")
# show interpolation
# functions for labels, as a function of alpha
main <- function(alpha) {if(alpha==0) "Original data"
    else if(alpha==1) "PCA scores"
    else paste(round(100*alpha,1), "% interpolation")}
xlab <- function(alpha) {if(alpha==0) "X"
    else if(alpha==1) "PCA.1"
    else paste("X +", alpha, "(X - PCA.1)")}
ylab <- function(alpha) {if(alpha==0) "Y"
```

```
    else if(alpha==1) "PCA.2"
    else paste("Y +", alpha, "(Y - PCA.2)")}
interpPCA <- function(XY, alpha = seq(0,1,.1)) {
    XY <- scale(XY, center=TRUE, scale=FALSE)
    if (is.null(rownames(XY))) rownames(XY) <- 1:nrow(XY)
    pca <- princomp(XY, cor=TRUE)
    scores <- pca$scores
    for (alp in alpha) {
        interpPlot(XY, scores, alp,
            pch=16,
            main = main(alp),
            xlab = xlab(alp),
            ylab = ylab(alp),
            ellipse=TRUE, ellipse.args=(list(levels=0.68, fill=TRUE, fill.alpha=(1-alp)/2)),
            abline=TRUE, id.n=2, id.cex=1.2, cex.lab=1.25, segments=TRUE)
        Sys.sleep(1)
    }
}
# show in R console
if(interactive()) {
interpPCA(XY)
}
## Not run:
library(animation)
saveGIF({
    interpPCA(XY, alpha <- seq(0,1,.1))},
    movie.name="outlier-demo.gif", ani.width=480, ani.height=480, interval=1.5)
## End(Not run)
```

    label.ellipse Label an ellipse
    
## Description

label.ellipse is used to a draw text label on an ellipse at its center or somewhere around the periphery in a very flexible way.

## Usage

label.ellipse(ellipse, label, col = "black", label.pos = NULL, xpd = TRUE, tweak $=0.5$ * c(strwidth("M"), strheight("M")), ...)

## Arguments

| ellipse | A two-column matrix of coordinates for the ellipse boundary |
| :--- | :--- |
| label | Character string to be used as the ellipse label |
| col | Label color |
| label.pos | Label position relative to the ellipse. See details |
| xpd | Should the label be allowed to extend beyond the plot limits? |
| tweak | A vector of two lengths used to tweak label positions |
| $\ldots$ | Other parameters passed to text, e.g., cex,... |

## Details

If label. pos=NULL, the function uses the sign of the correlation represented by the ellipse to determine a position at the top $(r>=0)$ or bottom $(r<0)$ of the ellipse. Integer values of 0 , $1,2,3$ and 4 , respectively indicate positions at the center, below, to the left of, above and to the right of the max/min coordinates of the ellipse. Label positions can also be specified as the corresponding character strings c("center", "bottom", "left", "top", "right"), or compass directions, $c(" C ", " S ", " W ", " N ", " E ")$, or Other integer label. pos values, 5: nrow(ellipse) are taken as indices of the row coordinates to be used for the ellipse label. Equivalently, label. pos can also be a fraction in $(0,1)$, interpreted as the fraction of the way around the unit circle, counterclockwise from the point $(1,0)$.

## Author(s)

Michael Friendly

## See Also

heplot

## Examples

```
circle <- function(center=c(0,0), radius=1, segments=60) {
    angles <- (0:segments)*2*pi/segments
    circle <- radius * cbind( cos(angles), sin(angles))
    t( c(center) + t( circle ))
}
label_demo <- function(ell) {
    plot(-2:2, -2:2, type="n", asp=1, main="label.pos values and points (0:60)")
    lines(ell, col="gray")
    points(0, 0, pch="+", cex=2)
    labs <- c("center", "bot", "left", "top", "right")
    for (i in 0:4) {
        label.ellipse(ell, label=paste(i, ":", labs[i+1]), label.pos = i)
    }
    for( i in 5*c(1,2, 4,5, 7,8, 10,11)) {
        points(ell[i,1], ell[i,2], pch=16)
```

```
        label.ellipse(ell, label=i, label.pos=i)
    }
}
circ <- circle(radius=1.8)
label_demo(circ)
ell <-circ %*% chol(matrix( c(1, .5, .5, 1), 2, 2))
label_demo(ell)
```

leveneTests Levene Tests of Homogeneity of Variances

## Description

This function extends leveneTest to a multivariate response setting. It performs the Levene test of homogeneity of variances for each of a set of response variables, and prints a compact summary.

## Usage

leveneTests(y, group, center = median, ...)

## Arguments

y A data frame or matrix of numeric response variables in a multivariate linear model.
group a vector or factor object giving the group for the corresponding elements of the rows of $y$
center The name of a function to compute the center of each group; mean gives the original Levene's (1960) test; the default, median, provides a more robust test suggested by Brown and Forsythe (1974).
... other arguments, passed to leveneTest

## Value

An object of classes "anova" and "data.frame", with one observation for each response variable in $y$.

## Author(s)

Michael Friendly

## References

Levene, H. (1960). Robust Tests for Equality of Variances. In Olkin, I. et al. (Eds.), Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling, Stanford University Press, 278292.

Brown, M. B. \& Forsythe, A. B. (1974). Robust Tests For Equality Of Variances Journal of the American Statistical Association, 69, 364-367.

## See Also

leveneTest, bartlettTests

## Examples

```
leveneTests(iris[,1:4], iris$Species)
# handle a 1-column response?
leveneTests(iris[,1, drop=FALSE], iris$Species)
data(Skulls, package="heplots")
leveneTests(Skulls[,-1], Skulls$epoch)
```

Calculate confidence interval for log determinant of covariance matrices

## Description

This function uses asymptotic results described by Cai et. al (2016), Theorem 1, to calculate approximate, normal theory confidence intervals (CIs) for the log determinant of one or more sample covariance matrices.
Their results are translated into a CI via the approximation

$$
\log \operatorname{det}(\widehat{\Sigma})-b i a s \pm z_{1-\alpha / 2} \times S E
$$

where $\widehat{\Sigma}$ is the sample estimate of a population covariance matrix, bias is a bias correction constant and $S E$ is a width factor for the confidence interval. Both bias and $S E$ are functions ofS the sample size, $n$ and number of variables, $p$.
This function is included here only to provide an approximation to graphical accuracy for use with Box's M test for equality of covariance matrices, boxM and its associated plot.boxM method.

## Usage

logdetCI (cov, $\mathrm{n}, \operatorname{conf}=0.95$, method $=1$, bias.adj $=$ TRUE)

## Arguments

cov
n

## conf

method Three methods are provided, based on Cai et. al Theorem 1 (method=1), Corollary 1 (method=2) and Corollary 2 (method=3), each with different bias and SE values.
bias.adj logical; set FALSE to exclude the bias correction term.

## Details

Cai et. al (2015) claim that their Theorem 1 holds with either $p$ fixed or $p(n)$ growing with $n$, as long as $p(n) \leq n$. Their Corollary 1 (method=2) is the special case when $p$ is fixed. Their Corollary 2 (method=3) is the special case when $0 \leq p / n<1$ is fixed.
The properties of this CI estimator are unknown in small to moderate sample sizes, but it seems to be the only one available. It is therefore experimental in this version of the package and is subject to change in the future.
The bias term offsets the confidence interval from the sample estimate of $\log \operatorname{det}(\widehat{\Sigma})$. When $p$ is large relative to $n$, the confidence interval may not overlap the sample estimate.
Strictly speaking, this estimator applies to the MLE of the covariance matrix $\widehat{\Sigma}$, i.e., using $n$ rather than $n-1$ in as the divisor. The factor $(n-1 / n)$ has not yet been taken into account here.

## Value

A data frame with one row for each covariance matrix. lower and upper are the boundaries of the confidence intervals. Other columns are logdet, bias, se.

## Author(s)

Michael Friendly

## References

Cai, T. T.; Liang, T. \& Zhou, H. H. (2015) Law of log determinant of sample covariance matrix and optimal estimation of differential entropy for high-dimensional Gaussian distributions. Journal of Multivariate Analysis, 137, 161-172. doi: 10.1016/j.jmva.2015.02.003

## See Also

boxM, plot.boxM

## Examples

```
data(iris)
iris.mod <- lm(as.matrix(iris[,1:4]) ~ iris$Species)
iris.boxm <- boxM(iris.mod)
cov <- c(iris.boxm$cov, list(pooled=iris.boxm$pooled))
n <- c(rep(50, 3), 150)
CI <- logdetCI( cov, n=n, conf=.95, method=1)
CI
plot(iris.boxm, xlim=c(-14, -8), main="Iris data, Box's M test", gplabel="Species")
arrows(CI$lower, 1:4, CI$upper, 1:4, lwd=3, angle=90, len=.1, code=3)
CI <- logdetCI( cov, n=n, conf=.95, method=1, bias.adj=FALSE)
CI
plot(iris.boxm, xlim=c(-14, -8), main="Iris data, Box's M test", gplabel="Species")
arrows(CI$lower, 1:4, CI$upper, 1:4, lwd=3, angle=90, len=.1, code=3)
```


## Description

This function is a convenience wrapper to mahalanobis offering also the possibility to calculate robust Mahalanobis squared distances using MCD and MVE estimators of center and covariance (from cov.rob)

## Usage

Mahalanobis(x, center, cov, method = c("classical", "mcd", "mve"), nsamp = "best", ...)

## Arguments

x
a numeric matrix or data frame with, say, $p$ columns
center mean vector of the data; if this and cov are both supplied, the function simply calls mahalanobis to calculate the result, ignoring the method argument.
cov covariance matrix ( $p x p$ ) of the data
method estimation method used for center and covariance, one of: "classical" (productmoment), "mcd" (minimum covariance determinant), or "mve" (minimum volume ellipsoid).
nsamp passed to cov.rob, just to make this argument explicit
... other arguments passed to cov. rob

## Details

Any missing data in a row of $x$ causes NA to be returned for that row.

## Value

A numeric vector of squared Mahalanobis distances corresponding to the rows of $x$.

## Author(s)

Michael Friendly

## See Also

mahalanobis, cov.rob

## Examples

```
summary(Mahalanobis(iris[, 1:4]))
summary(Mahalanobis(iris[, 1:4], method="mve"))
summary(Mahalanobis(iris[, 1:4], method="mcd"))
```

mark.H0 Mark a point null hypothesis in an HE plot

## Description

A utility function to draw and label a point in a 2D (or 3D) HE plot corresponding to a point null hypothesis being tested. This is most useful for repeated measure designs where null hypotheses for within-S effects often correspond to $(0,0)$.

## Usage

mark. $\mathrm{H} 0(\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=\mathrm{NULL}$, label, cex $=2$, $\mathrm{pch}=19$, col $=$ "green $3 "$, lty $=2$, pos $=2$ )

## Arguments

| $x$ | Horizontal coordinate for H0 |
| :--- | :--- |
| $y$ | Vertical coordinate for H0 |
| $z$ | z coordinate for H0. If not NULL, the function assumes that a heplot3d plot <br> has been drawn. |
| label | Text used to label the point. Defaults to expression(H[0]) in 2D plots. <br> cex |
| point and text size. For 3D plots, the function uses size=5*cex in a call to |  |
| points3d. |  |
| col | Plot character. Ignored for 3D plots. |
| $l$ ty | Color for text, character and lines |
| pos | Line type for vertical and horizontal reference lines. Not drawn if lty=0. <br> Position of text. Ignored for 3D plots |

## Value

None. Used for side effect of drawing on the current plot.

## Author(s)

Michael Friendly

## See Also

```
cross3d, ~~~
```


## Examples

```
Vocab.mod <- lm(cbind(grade8,grade9,grade10,grade11) ~ 1, data=VocabGrowth)
idata <-data.frame(grade=ordered(8:11))
heplot(Vocab.mod, type="III", idata=idata, idesign=~grade, iterm="grade",
main="HE plot for Grade effect")
mark.H0()
```


## Description

Scores for two groups of school children taught by different math teachers and tested for both basic math (BM) problems and solving word problems (WP).

## Usage

data("mathscore")

## Format

A data frame with 12 observations on the following 3 variables.
group a factor with levels 12
BM Basic Math score, a numeric vector
WP Word Problems score, a numeric vector

## Source

Fictitious data

## Examples

```
data(mathscore)
str(mathscore)
math.mod <- lm(cbind(BM, WP) ~ group, data=mathscore)
car::Anova(math.mod)
# scatterplot with data ellipses
car::scatterplot(WP ~ BM | group, data=mathscore,
ellipse=list(levels=0.68), smooth=FALSE, pch=c(15,16),
legend=list(coords = "topright"))
# HE plot
heplot(math.mod, fill=TRUE,
        cex=2, cex.lab=1.8,
xlab="Basic math", ylab="Word problems")
```


## Description

Male participants were shown a picture of one of three young women. Pilot work had indicated that the one woman was beautiful, another of average physical attractiveness, and the third unattractive. Participants rated the woman they saw on each of twelve attributes. These measures were used to check on the manipulation by the photo.
Then the participants were told that the person in the photo had committed a Crime, and asked to rate the seriousness of the crime and recommend a prison sentence, in Years.
Does attractiveness of the "defendant" influence the sentence or perceived seriousness of the crime? Does attractiveness interact with the nature of the crime?

## Usage

data(MockJury)

## Format

A data frame with 114 observations on the following 17 variables.
Attr Attractiveness of the photo, a factor with levels Beautiful Average Unattractive
Crime Type of crime, a factor with levels Burglary (theft of items from victim's room) Swindle (conned a male victim)
Years length of sentence given the defendant by the mock juror subject
Serious a rating of how serious the subject thought the defendant's crime was
exciting rating of the photo for 'exciting'
calm rating of the photo for 'calm'
independent rating of the photo for 'independent'
sincere rating of the photo for 'sincere'
warm rating of the photo for 'warm'
phyattr rating of the photo for 'physical attractiveness'
sociable rating of the photo for 'exciting'
kind rating of the photo for 'kind'
intelligent rating of the photo for 'intelligent'
strong rating of the photo for 'strong'
sophisticated rating of the photo for 'sophisticated'
happy rating of the photo for 'happy'
ownPA self-rating of the subject for 'physical attractiveness'

## Source

From Dr. Wuensch's StatData Page, http://core.ecu.edu/psyc/wuenschk/StatData/PLASTER. dat

## References

Data from the thesis by Plaster, M. E. (1989). Inmates as mock jurors: The effects of physical attractiveness upon juridic decisions. M.A. thesis, Greenville, NC: East Carolina University.

## Examples

```
# manipulation check: test ratings of the photos classified by Attractiveness
jury.mod1 <- lm( cbind(phyattr, happy, independent, sophisticated) ~ Attr, data=MockJury)
Anova(jury.mod1, test="Roy")
##
## Type II MANOVA Tests: Roy test statistic
## Df test stat approx F num Df den Df Pr(>F)
## Attr 2 1.7672 48.156 4 109 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
heplot(jury.mod1, main="HE plot for manipulation check")
pairs(jury.mod1)
if (require(candisc)) {
jury.can <- candisc(jury.mod1)
jury.can
heplot(jury.can, main="Canonical HE plot")
}
# influence of Attr of photo and nature of crime on Serious and Years
jury.mod2 <- lm( cbind(Serious, Years) ~ Attr * Crime, data=MockJury)
Anova(jury.mod2, test="Roy")
heplot(jury.mod2)
# stepdown test (ANCOVA), controlling for Serious
jury.mod3 <- lm( Years ~ Serious + Attr * Crime, data=MockJury)
Anova(jury.mod3)
# need to consider heterogeneous slopes?
jury.mod4 <- lm( Years ~ Serious * Attr * Crime, data=MockJury)
anova(jury.mod3, jury.mod4)
```


## Description

The primary purpose of the study (Hartman, 2016, Heinrichs etal. (2015)) was to evaluate patterns and levels of performance on neurocognitive measures among individuals with schizophrenia and schizoaffective disorder using a well-validated, comprehensive neurocognitive battery specifically designed for individuals with psychosis (Heinrichs et al. (2008))
The main interest was in determining how well these measures distinguished among all groups and whether there were variables that distinguished between the schizophrenia and schizoaffective groups.

## Usage <br> data("NeuroCog")

## Format

A data frame with 242 observations on the following 10 variables.
Dx Diagnostic group, a factor with levels Schizophrenia Schizoaffective Control
Speed Speed of processing domain T score, a numeric vector
Attention Attention/Vigilance Domain T score, a numeric vector
Memory Working memory a numeric vector
Verbal Verbal Learning Domain T score, a numeric vector
Visual Visual Learning Domain T score, a numeric vector
ProbSolv Reasoning/Problem Solving Domain T score, a numeric vector
SocialCog Social Cognition Domain T score, a numeric vector
Age Subject age, a numeric vector
Sex Subject gender, a factor with levels Female Male

## Details

Neurocognitive function was assessed using the MATRICS Consensus Cognitive Battery (MCCB; Nuechterlein et al., 2008). The MCCB consists of 10 individually administered tests that measure cognitive performance in seven domains: speed of processing, attention/vigilance, working memory, verbal learning, visual learning, reasoning and problem solving, and social cognition.
The clinical sample comprised 116 male and female patients who met the following criteria: 1) a diagnosis of schizophrenia $(\mathrm{n}=70)$ or schizoaffective disorder $(\mathrm{n}=46)$ confirmed by the Structured Clinical Interview for DSM-IV-TR Axis I Disorders; 2) outpatient status; 3) a history free of developmental or learning disability; 4) age $18-65$; 5) a history free of neurological or endocrine disorder; and 6) no concurrent DSM-IV-TR diagnosis of substance use disorder.

Non-psychiatric control participants $(\mathrm{n}=146)$ were screened for medical and psychiatric illness and history of substance abuse. Patients were recruited from three outpatient clinics in Hamilton, Ontario, Canada. Control participants were recruited through local newspaper and online classified advertisements for paid research participation.

## Source

Hartman, L. I. (2016). Schizophrenia and Schizoaffective Disorder: One Condition or Two? Unpublished PhD dissertation, York University.
Heinrichs, R.W., Pinnock, F., Muharib, E., Hartman, L.I., Goldberg, J.O., \& McDermid Vaz, S. (2015). Neurocognitive normality in schizophrenia revisited. Schizophrenia Research: Cognition, 2 (4), 227-232. doi: 10.1016/j.scog. 2015.09.001

## References

Heinrichs, R. W., Ammari, N., McDermid Vaz, S. \& Miles, A. (2008). Are schizophrenia and schizoaffective disorder neuropsychologically distinguishable? Schizophrenia Research, 99, 149154.

Nuechterlein K.H., Green M.F., Kern R.S., Baade L.E., Barch D., Cohen J., Essock S., Fenton W.S., Frese F.J., Gold J.M., Goldberg T., Heaton R., Keefe R.S.E., Kraemer H., Mesholam-Gately R., Seidman L.J., Stover E., Weinberger D.R., Young A.S., Zalcman S., Marder S.R. (2008) The MATRICS Consensus Cognitive Battery, Part 1: Test selection, reliability, and validity. American Journal of Psychiatry, 165 (2), 203-213. https: //pubmed.ncbi .nlm.nih.gov/18172019/.

## Examples

```
data(NeuroCog)
NC.mlm <- lm(cbind( Speed, Attention, Memory, Verbal, Visual, ProbSolv) ~ Dx,
                                    data=NeuroCog)
Anova(NC.mlm)
# test contrasts
contrasts(NeuroCog$Dx)
print(linearHypothesis(NC.mlm, "Dx1"), SSP=FALSE)
print(linearHypothesis(NC.mlm, "Dx2"), SSP=FALSE)
# pairwise HE plots
pairs(NC.mlm, var.cex=1.5)
# canonical discriminant analysis
if (require(candisc)) {
    NC.can <- candisc(NC.mlm)
    NC.can
    plot(NC.can, ellipse=TRUE, rev.axes=c(TRUE,FALSE), pch=c(7,9,10))
}
```


## NLSY

 National Longitudinal Survey of Youth Data
## Description

The dataset come from a small random sample of the U.S. National Longitudinal Survey of Youth.

## Usage

data(NLSY)

## Format

A data frame with 243 observations on the following 6 variables.
math Math achievement test score
read Reading achievement test score
antisoc score on a measure of child's antisocial behavior, $0: 6$
hyperact score on a measure of child's hyperactive behavior, 0:5
income yearly income of child's father
educ years of education of child's father

## Details

In this dataset, math and read scores are taken at the outcome variables. Among the remaining predictors, income and educ might be considered as background variables necessary to control for. Interest might then be focused on whether the behavioural variables antisoc and hyperact contribute beyond that.

## Source

This dataset was derived from a larger one used by Patrick Curran at the 1997 meeting of the Society for Research on Child Development (SRCD). A description now only exists on the WayBack Machine, http://web.archive.org/web/20050404145001/http://www.unc.edu/~curran/example. html.
More details are available at http://web.archive.org/web/20060830061414/http://www.unc. edu/~curran/srcd-docs/srcdmeth.pdf.

## Examples

```
data(NLSY)
#examine the data
scatterplotMatrix(NLSY, smooth=FALSE)
# test control variables by themselves
# ------------------------------------
mod1 <- lm(cbind(read,math) ~ income+educ, data=NLSY)
Anova(mod1)
heplot(mod1, fill=TRUE)
# test of overall regression
coefs <- rownames(coef(mod1))[-1]
linearHypothesis(mod1, coefs)
heplot(mod1, fill=TRUE, hypotheses=list("Overall"=coefs))
```

```
# additional contribution of antisoc + hyperact over income + educ
# ---------------------------------------------------------------
mod2 <- lm(cbind(read,math) ~ antisoc + hyperact + income + educ, data=NLSY)
Anova(mod2)
coefs <- rownames(coef(mod2))[-1]
heplot(mod2, fill=TRUE, hypotheses=list("Overall"=coefs, "mod2|mod1"=coefs[1:2]))
linearHypothesis(mod2, coefs[1:2])
heplot(mod2, fill=TRUE, hypotheses=list("mod2|mod1"=coefs[1:2]))
```

Oslo Oslo Transect Subset Data

## Description

The Oslo data set contains chemical concentrations of 332 samples of different plant species collected along a 120 km transect running through the city of Oslo, Norway. It is a subset of the OsloTransect data provided by the rrcov package.

## Usage

data(Oslo)

## Format

A data frame with 332 observations on the following 14 variables.
site transect site ID, a factor with levels 102103104105106107108109111112113114115 116117118119121122123124125126127128129131132133134135136138139 141142143144
XC X coordinate, a numeric vector
YC Y coordinate, a numeric vector
forest forest type, a factor with levels birspr mixdec pine sprbir sprpin spruce
weather weather type, a factor with levels cloud moist nice rain
litho lithological type, a factor with levels camsed (Cambro-Silurian sedimentary), gneis_o (Precambrian gneisses - Oslo), gneis_r (- Randsfjord), magm (Magmatic rocks)
altitude altitude, a numeric vector
Cu Copper, a numeric vector
Fe Iron, a numeric vector
K Potassium, a numeric vector
Mg Magnesium, a numeric vector
Mn Manganese, a numeric vector
P Lead, a numeric vector
Zn Zinc, a numeric vector

## Details

The OsloTransect contains 360 observations, with 9 observations per site. Only 7 chemical elements were retained from the 25 contained in the OsloTransect data, and these were all logtransformed, following Todorov and Filzmoser (2009).

Only complete cases on these variables were retained, and two lithological types of low frequency were removed, leaving 332 observations.

## Source

Reimann, C., Arnoldussen, A., Boyd, R., Finne, T.E., Koller, F., Nordgulen, Oe., And Englmaier, P. (2007) Element contents in leaves of four plant species (birch, mountain ash, fern and spruce) along anthropogenic and geogenic concentration gradients, The Science of the Total Environment, 377, 416-433.

## References

Todorov V. and Filzmoser P. (2009) Robust statistic for the one-way MANOVA, submitted to the Journal of Environmetrics.

## Examples

```
data(Oslo)
table(Oslo$litho)
Oslo.mod <- lm(cbind(Cu, K, Mg, Mn, P, Zn) ~ litho, data=Oslo)
Anova(Oslo.mod)
heplot(Oslo.mod, var=c("Cu", "Mn"))
pairs(Oslo.mod)
## Not run:
if(require(candisc)) {
        Oslo.can <- candisc(Oslo.mod)
    Oslo.can
    heplot(Oslo.can)
        if(requireNamespace("rgl")){
            heplot3d(Oslo.can, shade=TRUE, wire=FALSE, alpha=0.5, var.col="red")
    }
}
## End(Not run)
```


## Description

The function (in the form of an mlm method for the generic pairs function) constructs a "matrix" of pairwise HE plots (see heplot) for a multivariate linear model.

```
Usage
## S3 method for class 'mlm'
pairs(x, variables, var.labels, var.cex=2,
    type = c("II", "III", "2", "3"),
idata=NULL, idesign=NULL, icontrasts=NULL, imatrix=NULL, iterm=NULL, manova,
    offset.axes = 0.05, digits = getOption("digits") - 1, fill=FALSE, fill.alpha=0.3, ...)
```


## Arguments

| x | an object of class mlm. |
| :---: | :---: |
| variables | indices or names of the three of more response variables to be plotted; defaults to all of the responses. |
| var.labels | labels for the variables plotted in the diagonal panels; defaults to names of the response variables. |
| var.cex | character expansion for the variable labels. |
| type | "type" of sum-of-squares-and-products matrices to compute; one of "II", "III", " 2 ", or " 3 ", where "II" is the default (and " 2 " is a synonym). |
| idata | an optional data frame giving a factor or factors defining the intra-subject model for multivariate repeated-measures data. See Details of Anova for an explanation of the intra-subject design and for further explanation of the other arguments relating to intra-subject factors. |
| idesign | a one-sided model formula using the "data" in idata and specifying the intrasubject design for repeated measure models. |
| icontrasts | names of contrast-generating functions to be applied by default to factors and ordered factors, respectively, in the within-subject "data"; the contrasts must produce an intra-subject model matrix in which different terms are orthogonal. The default is c("contr.sum", "contr.poly"). |
| imatrix | In lieu of idata and idesign, you can specify the intra-subject design matrix directly via imatrix, in the form of list of named elements. Each element gives the columns of the within-subject model matrix for an intra-subject term to be tested, and must have as many rows as there are responses; the columns of the within-subject model matrix for different terms must be mutually orthogonal. This functionality requires car version 2.0 or later. |
| iterm | For repeated measures designs, you must specify one intra-subject term (a character string) to select the SSPE (E) matrix used in the HE plot. Hypothesis terms plotted include the iterm effect as well as all interactions of iterm with terms. |
| manova | optional Anova.mlm object for the model; if absent a MANOVA is computed. Specifying the argument can therefore save computation in repeated calls. |
| offset.axes | proportion to extend the axes in each direction; defaults to 0.05. |
| digits | number of significant digits in axis end-labels; taken from the "digits" option. |

fill A logical vector indicating whether each ellipse should be filled or not. The first value is used for the error ellipse, the rest - possibly recycled - for the hypothesis ellipses; a single fill value can be given. Defaults to FALSE for backward compatibility. See Details of heplot
fill.alpha Alpha transparency for filled ellipses, a numeric scalar or vector of values within $[0,1]$, where 0 means fully transparent and 1 means fully opaque. Defaults to 0.3.
... arguments to pass down to heplot, which is used to draw each panel of the display.

## Author(s)

Michael Friendly

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples Journal of Statistical Software, 17(6), 1-42. https://www. jstatsoft.org/v17/i06/
Friendly, M. (2007). HE plots for Multivariate General Linear Models. Journal of Computational and Graphical Statistics, 16(2) 421-444. http://datavis.ca/papers/jcgs-heplots.pdf

## See Also

heplot, heplot3d

## Examples

\# ANCOVA, assuming equal slopes
rohwer. mod <- lm(cbind(SAT, PPVT, Raven) $\sim$ SES + n + s + ns + na + ss, data=Rohwer)
\# View all pairs, with ellipse for all 5 regressors
pairs(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")))

## Description

The data, from an exercise given by Meyers et al. (2006) relates to 60 fathers assessed on three subscales of a Perceived Parenting Competence Scale. The fathers were selected from three groups: (a) fathers of a child with no disabilities; (b) fathers with a physically disabled child; (c) fathers with a mentally disabled child.

## Usage

```
    data("Parenting")
```


## Format

A data frame with 60 observations on the following 4 variables.
group a factor with levels Normal Physical Disability Mental Disability
caring caretaking responsibilities, a numeric vector
emotion emotional support provided to the child, a numeric vector
play recreational time spent with the child, a numeric vector

## Details

The scores on the response variables are discrete.

## Source

Meyers, L. S., Gamst, G, \& Guarino, A. J. (2006). Applied Multivariate Research: Design and Interpretation, Thousand Oaks, CA: Sage Publications, https://studysites.sagepub.com/amrStudy/, Exercises 10B.

## Examples

```
data(Parenting)
require(car)
# fit the MLM
parenting.mod <- lm(cbind(caring, emotion, play) ~ group, data=Parenting)
Anova(parenting.mod)
# Box's M test
boxM(parenting.mod)
plot(boxM(parenting.mod))
parenting.mod <- lm(cbind(caring, emotion, play) ~ group, data=Parenting)
Anova(parenting.mod)
# test contrasts
print(linearHypothesis(parenting.mod, "group1"), SSP=FALSE)
print(linearHypothesis(parenting.mod, "group2"), SSP=FALSE)
heplot(parenting.mod)
# display tests of contrasts
hyp <- list("N:MP" = "group1", "M:P" = "group2")
heplot(parenting.mod, hypotheses=hyp)
# make a prettier plot
heplot(parenting.mod, hypotheses=hyp, asp=1,
            fill=TRUE, fill.alpha=c(0.3,0.1),
            col=c("red", "blue"),
            lty=c(0,0,1,1), label.pos=c(1,1,3,2),
            cex=1.4, cex.lab=1.4, lwd=3)
pairs(parenting.mod, fill=TRUE, fill.alpha=c(0.3, 0.1))
```

```
    ## Not run:
    heplot3d(parenting.mod, wire=FALSE)
    ## End(Not run)
```

    Plastic Plastic Film Data
    
## Description

An experiment was conducted to determine the optimum conditions for extruding plastic film. Three responses were measured in relation to two factors, rate of extrusion and amount of an additive.

## Usage

data(Plastic)

## Format

A data frame with 20 observations on the following 5 variables.
tear a numeric vector: tear resistance
gloss a numeric vector: film gloss
opacity a numeric vector: film opacity
rate a factor representing change in the rate of extrusion with levels Low ( $-10 \%$ ), High ( $10 \%$ )
additive a factor with levels Low (1.0\%), High (1.5\%)

## Source

Johnson, R.A. \& Wichern, D.W. (1992). Applied Multivariate Statistical Analysis, 3rd ed., PrenticeHall. Example 6.12 (p. 266).

## References

Krzanowski, W. J. (1988). Principles of Multivariate Analysis. A User's Perspective. Oxford. (p. 381)

## Examples

```
str(Plastic)
plastic.mod <- lm(cbind(tear, gloss, opacity) ~ rate*additive, data=Plastic)
Anova(plastic.mod)
```

pairs(plastic.mod)

## Description

This function creates a simple dot chart showing the contributions (log determinants) of the various groups to Box's M test for equality of covariance matrices.
Because Box's M test is based on a specific function (log determinant) of the covariance matrices in the groups compared to the pooled covariance matrix, this function also also allow plots of other measures based on the eigenvalues of these covariance matrices.

## Usage

```
## S3 method for class 'boxM'
    plot(x,
```

```
gplabel = NULL,
```

gplabel = NULL,
which = c("logDet", "product", "sum", "precision", "max"),
which = c("logDet", "product", "sum", "precision", "max"),
log = which=="product",
log = which=="product",
pch = c(16, 15),
pch = c(16, 15),
cex = c(2, 2.5),
cex = c(2, 2.5),
col = c("blue", "red"),
col = c("blue", "red"),
rev = FALSE,
rev = FALSE,
xlim,
xlim,
conf=0.95, method=1, bias.adj=TRUE, lwd=2,
conf=0.95, method=1, bias.adj=TRUE, lwd=2,
...)

```
...)
```


## Arguments

## x

A "boxM" object resulting from boxM

## gplabel

which
$\log \quad l o g i c a l$; if TRUE, the log of the measure is plotted. The default, which=="product", produces a plot equivalent to the plot of "logDet".
pch a vector of two point symbols to use for the individual groups and the pooled data, respectively
cex character size of point symbols, a vector of length two for groups and pooled data, respectively
col colors for point symbols, a vector of length two for the groups and the pooled data
rev
xlim
conf
logical; if TRUE, the order of the groups is reversed on the vertical axis.
$x$ limits for the plot
coverage for approximate confidence intervals, $0<=$ conf $<1$; use conf=0 to suppress these

| method | confidence interval method; see logdetCI |
| :--- | :--- |
| bias.adj | confidence interval bias adjustment; see logdetCI |
| lwd | line width for confidence interval |
| $\ldots$ | Arguments passed down to dotchart. |

## Details

Confidence intervals are only available for the default Box M test, using which="logDet".

## Author(s)

Michael Friendly

## See Also

boxM, logdetCI
dotchart

## Examples

```
# Iris data
res <- boxM(iris[, 1:4], iris[, "Species"])
plot(res, gplabel="Species")
# Skulls data
skulls.mod <- lm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
skulls.boxm <- boxM(skulls.mod)
plot(skulls.boxm, gplabel="Epoch")
plot(skulls.boxm, gplabel="Epoch", bias.adj=FALSE)
# other measures
    plot(skulls.boxm, which="product", gplabel="Epoch", xlim=c(10,14))
    plot(skulls.boxm, which="sum", gplabel="Epoch")
    plot(skulls.boxm, which="precision", gplabel="Epoch")
    plot(skulls.boxm, which="max", gplabel="Epoch")
```

    plot.robmlm
    
## Description

Creates an index plot of the observation weights assigned in the last iteration of robmlm. Observations with low weights have large residual squared distances and are potential multivariate outliers with respect to the fitted model.

## Usage

```
\#\# S3 method for class 'robmlm'
plot(x, labels,
    id.weight \(=0.7\), id.pos \(=4\),
    pch = 19,
    col = palette()[1], cex = par("cex"),
    segments = FALSE,
    xlab = "Case index", ylab = "Weight in robust MANOVA", ...)
```


## Arguments

| x | A "robmlm" object |
| :--- | :--- |
| labels | Observation labels; if not specified, uses rownames from the original data |
| id.weight | Threshold for identifying observations with small weights |
| id.pos | Position of observation label relative to the point |
| pch | Point symbol(s); can be a vector of length equal to the number of observations <br> in the data frame |
| col | Point color(s) |
| cex | Point character size(s) |
| segments | logical; if TRUE, draw line segments from 1.o down to the point |
| xlab | x axis label |
| ylab | y axis label |
| $\ldots$ | other arguments passed to plot |

## Value

Returns invisibly the weights for the observations labeled in the plot

## Author(s)

Michael Friendly

## See Also

robmlm

## Examples

```
data(Skulls)
sk.rmod <- robmlm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
plot(sk.rmod, col=Skulls$epoch)
axis(side=3, at=15+seq(0,120,30), labels=levels(Skulls$epoch), cex.axis=1)
# Pottery data
pottery.rmod <- robmlm(cbind(Al,Fe,Mg,Ca,Na)~Site, data=Pottery)
plot(pottery.rmod, col=Pottery$Site, segments=TRUE)
```

\# SocialCog data
data(SocialCog)
SC.rmod <- robmlm(cbind( MgeEmotions, ToM, ExtBias, PersBias) ~ Dx, data=SocialCog)
plot(SC.rmod, col=SocialCog\$Dx, segments=TRUE)
Pottery2 Chemical Analysis of Romano-British Pottery

## Description

Results of chemical analyses of 48 specimens of Romano-British pottery published by Tubb et al. (1980). The numbers are the percentage of various metal oxides found in each sample for elements of concentrations greater than $0.01 \%$. This is the original data set from Tubb et al. (1980), in contrast to Pottery.

## Usage

data(Pottery2)

## Format

A data frame with 48 observations on the following 12 variables.
Region a factor with levels Gl NF Wales
Site a factor with levels AshleyRails Caldicot Gloucester IsleThorns Llanedryn
Kiln a factor with levels 12345
Al amount of aluminum oxide, $\mathrm{Al}_{2} \mathrm{O}_{3}$
Fe amount of iron oxide, $\mathrm{Fe}_{2} \mathrm{O}_{3}$
Mg amount of magnesium oxide, MgO
Ca amount of calcium oxide, CaO
Na amount of sodium oxide, $\mathrm{Na}_{2} \mathrm{O}$
K amount of potassium oxide, $\mathrm{K}_{2} \mathrm{O}$
Ti amount of titanium oxide, $\mathrm{TiO}_{2}$
Mn amount of manganese oxide, MnO
Ba amount of BaO

## Details

The specimens are identified by their rownames in the data frame. Kiln indicates at which kiln site the pottery was found; Site gives the location names of those sites. The kiln sites come from three Regions, ("Gl" $=1$, "Wales" $=(2,3), " N F "=(4,5)$ ), where the full names are "Gloucester", "Wales", and "New Forrest".
The variable Kiln comes pre-supplied with contrasts to test interesting hypotheses related to Site and Region.

## Source

Slightly modified from files found at: http://people. tamu.edu/~dcarlson/quant/data/RBPottery. html

## References

Baxter, M. J. 2003. Statistics in Archaeology. Arnold, London.
Tubb, A., A. J. Parker, and G. Nickless. 1980. The Analysis of Romano-British Pottery by Atomic Absorption Spectrophotometry. Archaeometry, 22, 153-171.

## See Also

Pottery for the related (subset) data set

## Examples

```
data(Pottery2)
# contrasts for Kiln correspond to between Region [,1:2] and within Region [,3:4]
contrasts(Pottery2$Kiln)
pmod <-lm(cbind(Al,Fe,Mg,Ca,Na,K,Ti,Mn,Ba)~Kiln, data=Pottery2)
Anova(pmod)
# extract coefficient names for linearHypotheses
coefs <- rownames(coef(pmod))[-1]
# test differences among regions
linearHypothesis(pmod, coefs[1:2])
# test differences within regions B, C
linearHypothesis(pmod, coefs[3:4])
heplot(pmod, fill=c(TRUE,FALSE), hypotheses=list("Region" =coefs[1:2], "WithinBC"=coefs[3:4]))
# all pairwise views; note that Ba shows no effect
pairs(pmod, fill=c(TRUE,FALSE))
# canonical view, via candisc::heplot
if (require(candisc)) {
# canonical analysis: how many dimensions?
```

```
    (pcan <- candisc(pmod))
    heplot(pcan, scale=18, fill=c(TRUE,FALSE), var.col="darkgreen", var.lwd=2, var.cex=1.5)
    ## Not run:
    heplot3d(pcan, scale=8)
    ## End(Not run)
}
```

Probe Response Speed in a Probe Experiment

## Description

Data from a probe experiment testing whether immediate memory for sentences is influenced by the phrase structure of the sentence. The data sets come from Timm (1975), Ex. 3.14 and Ex. 3.16 (p.244)

Procedure: Subjects listened to tape-recorded sentences. Each sentence was followed by a "probe word" from one of 5 positions within the sentence. The subject had to respond with the word which immediately followed the probe word in the sentence. The dependent measure is response speed $=$ $\mathrm{k}(1 /$ reaction time $)$.

Sample sentence:

* The tall man met the young girl who got the new hat.
Pos'ns:
Function:
F ADJ1 SUBJ

In Probe2, there are two groups of subjects, pre-selected on a test of short term memory.

## Usage

data(Probe1) data(Probe2)

## Format

Probe1: A data frame with 11 observations on the following 5 variables.
p 1 speed at position 1
p2 speed at position 2
p3 speed at position 3
p4 speed at position 4
p5 speed at position 5

Probe2: A data frame with 20 observations on the following 6 variables.
stm Short term memory capacity: a factor with levels High Low
p1 speed at position 1
p2 speed at position 2
p3 speed at position 3
p4 speed at position 4
p5 speed at position 5

## Details

These data sets (fictitious) are used as examples of single-sample and two-sample profile analysis or simple repeated measure designs with structured contrasts.

## Source

Timm, N. (1975) Multivariate analysis, with applications in education and psychology Brooks/Cole.

## Examples

```
data(Probe1)
boxplot(Probe1)
pmod1 <- lm(cbind(p1,p2,p3,p4,p5) ~ 1, data=Probe1)
idata <- data.frame(position=factor(1:5))
library(car)
(pmod1.aov <- Anova(pmod1, idata=idata, idesign=~position))
# using default contrasts (p5 as reference level)
heplot(pmod1, manova=pmod1.aov, iterm="position", type="III", idata=idata, idesign=~position)
pairs(pmod1, manova=pmod1.aov, iterm="position", type="III", idata=idata, idesign=~position)
# contrasts for substantative hypotheses regarding
# sentence position effects
C <- matrix(c(
1, 1, -1, -1, 0,
1, -1, 1, -1, 0,
1, -1, -1, 1, 0,
1, 1, 1, 1, 4), 5, 4)
rownames(C) <- paste("p", 1:5, sep="")
colnames(C) <- c("SubPred", "AdjNoun", "SPxAN", "RelPN")
contrasts(idata$position)<- C
(pmod1.aov <- Anova(pmod1, idata=idata, idesign=~position))
heplot(pmod1, manova=pmod1.aov, iterm="position", type="III", idata=idata, idesign=~position)
pairs(pmod1, manova=pmod1.aov, iterm="position", type="III", idata=idata, idesign=~position)
```


## Description

The data are from a study of weight gain, where investigators randomly assigned 30 rats to three treatment groups: treatment 1 was a control (no additive); treatments 2 and 3 consisted of two different additives (thiouracil and thyroxin respectively) to the rats drinking water. Weight was measured at baseline (week 0) and at weeks $1,2,3$, and 4 . Due to an accident at the beginning of the study, data on 3 rats from the thyroxin group are unavailable.

## Usage

data(RatWeight)

## Format

A data frame with 27 observations on the following 6 variables.
trt a factor with levels Control Thiouracil Thyroxin
wt0 Weight at Week 0 (baseline weight)
wt1 Weight at Week 1
wt2 Weight at Week 2
wt3 Weight at Week 3
wt4 Weight at Week 4

## Details

The trt factor comes supplied with contrasts comparing Control to each of Thiouracil and Thyroxin.

## Source

Originally from Box (1950), Table D (page 389), where the values for weeks 1-4 were recorded as the gain in weight for that week.

Fitzmaurice, G. M. and Laird, N. M. and Ware, J. H (2004). Applied Longitudinal Analysis, New York, NY: Wiley-Interscience. https://rdrr.io/rforge/ALA/.

## References

Box, G.E.P. (1950). Problems in the analysis of growth and wear curves. Biometrics, 6, 362-389.
Friendly, Michael (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04.

## Examples

```
data(RatWeight)
contrasts(RatWeight$trt)
rat.mod <- lm(cbind(wt0, wt1, wt2, wt3, wt4) ~ trt, data=RatWeight)
rat.mod
idata <- data.frame(week = ordered(0:4))
Anova(rat.mod, idata=idata, idesign=~week, test="Roy")
# quick look at between group effects
pairs(rat.mod)
# between-S, baseline & week 4
heplot(rat.mod, col=c("red", "blue", "green3", "green3"),
variables=c(1,5),
hypotheses=c("trt1", "trt2"),
main="Rat weight data, Between-S effects")
# within-S
heplot(rat.mod, idata=idata, idesign=~week, iterm="week",
col=c("red", "blue", "green3"),
# hypotheses=c("trt1", "trt2"),
main="Rat weight data, Within-S effects")
```

ReactTime Reaction Time Data

## Description

Data from Maxwell and Delaney (1990, p. 497) representing the reaction times of 10 subjects in some task where visual stimuli are tilted at 0,4 , and 8 degrees; with noise absent or present. Each subject responded to 3 tilt $\times 2$ noise $=6$ conditions. The data thus comprise a repeated measure design with two within-S factors.

## Usage

data(ReactTime)

## Format

A data frame with 10 observations giving the reaction time for the 6 conditions.
deg0NA a numeric vector
deg4NA a numeric vector
deg8NA a numeric vector
deg0NP a numeric vector
deg4NP a numeric vector
deg8NP a numeric vector
robmlm

## Source

Baron, J. and Li, Y. (2003). Notes on the use of R for psychology experiments and questionnaires, https://cran.r-project.org/doc/contrib/Baron-rpsych.pdf

## References

Michael Friendly (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04.
Maxwell, S. E. \& Delaney, H. D. (1990). Designing Experiments and Analyzing Data: A model comparison perspective. Pacific Grove, CA: Brooks/Cole.

## Examples

```
data(ReactTime)
(RT.mod <- lm(as.matrix(ReactTime)~1))
# within-S factors
within <- expand.grid(tilt=ordered(c(0,4,8)), noise=c("NA", "NP"))
Anova(RT.mod, idata=within, idesign=~tilt * noise)
heplot(RT.mod, idata=within, idesign=~tilt * noise, iterm="tilt")
# plotting means and std errors directly
levels <- expand.grid(Tilt=c(0,4,8), noise=c("NA", "NP"))
(means.df <- data.frame(levels, mean=colMeans(ReactTime), se=sqrt(diag(var(ReactTime)))/9))
with(means.df, {
plot(Tilt, mean, type="n", main="Reaction Time data", xlab="Tilt", ylab="Reaction time")
colors <- rep(c("red", "blue"), each=3)
pts <- rep(c(15, 16), each=3)
lines(Tilt[1:3], mean[1:3], col="red", lwd=2)
lines(Tilt[4:6], mean[4:6], col="blue", lwd=2)
points(Tilt, mean, pch=pts, col=colors, cex=1.2)
arrows(Tilt, mean-se, Tilt, mean+se, angle=90, code=3,
col=colors, len=.05, lwd=2)
# labels at last point, in lieu of legend
text(Tilt[3], mean[3]-10, labels="NA", col="red", pos=1)
text(Tilt[6], mean[6]-10, labels="NP", col="blue", pos=1)
}
)
```


## Description

Fit a multivariate linear model by robust regression using a simple M estimator.
These S3 methods are designed to provide a specification of a class of robust methods which extend mlms , and are therefore compatible with other mlm extensions, including Anova and heplot.

## Usage

```
robmlm(X, ...)
## S3 method for class 'formula'
robmlm(formula, data, subset, weights, na.action,
model = TRUE, contrasts = NULL, ...)
## Default S3 method:
robmlm(X, Y, w,
P = 2 * pnorm(4.685, lower.tail = FALSE), tune, max.iter = 100,
psi = psi.bisquare, tol = 1e-06, initialize, verbose = FALSE, ...)
## S3 method for class 'robmlm'
print(x, ...)
## S3 method for class 'summary.robmlm'
print(x, ...)
## S3 method for class 'robmlm'
summary(object, ...)
```


## Arguments

| formula | a formula of the form cbind $(\mathrm{y} 1, \mathrm{y} 2, \ldots) \sim \mathrm{x} 1+\mathrm{x} 2+\ldots$ |
| :--- | :--- |
| data | a data frame from which variables specified in formula are preferentially to be <br> taken. |
| subset | An index vector specifying the cases to be used in fitting. |
| weights | a vector of prior weights for each case. |
| na.action | A function to specify the action to be taken if NAs are found. The 'factory-fresh' <br> default action in R is na. omit, and can be changed by options (na. action=). |
| model | should the model frame be returned in the object? |
| contrasts | optional contrast specifications; see lm for details. <br> other arguments, passed down. In particular relevant control arguments can be <br> passed to the to the robmlm. default method. |
| X | for the default method, a model matrix, including the constant (if present) |

robmlm
$Y$ for the default method, a response matrix
w prior weights
$P \quad$ two-tail probability, to find cutoff quantile for chisq (tuning constant); default is set for bisquare weight function
tune tuning constant (if given directly)
max.iter maximum number of iterations
psi robustness weight function; psi.bisquare is the default
tol convergence tolerance, maximum relative change in coefficients
initialize modeling function to find start values for coefficients, equation-by-equation; if absent WLS (lm.wfit) is used
verbose show iteration history? (TRUE or FALSE)
$x \quad$ a robmlm object
object a robmlm object

## Details

Fitting is done by iterated re-weighted least squares (IWLS), using weights based on the Mahalanobis squared distances of the current residuals from the origin, and a scaling (covariance) matrix calculated by cov. trob. The design of these methods were loosely modeled on rlm.

An internal vcov.mlm function is an extension of the standard vcov method providing for observation weights.

## Value

An object of class "robmlm" inheriting from c("mlm", "lm").
This means that the returned "robmlm" contains all the components of "mlm" objects described for lm, plus the following:

| weights | final observation weights |
| :--- | :--- |
| iterations | number of iterations |
| converged | logical: did the IWLS process converge? |

The generic accessor functions coefficients, effects, fitted.values and residuals extract various useful features of the value returned by robmlm.

## Author(s)

John Fox; packaged by Michael Friendly

## References

A. Marazzi (1993) Algorithms, Routines and S Functions for Robust Statistics. Wadsworth \& Brooks/Cole.

## See Also

rlm, cov.trob

## Examples

```
##############
# Skulls data
# make shorter labels for epochs and nicer variable labels in heplots
Skulls$epoch <- factor(Skulls$epoch, labels=sub("c","",levels(Skulls$epoch)))
# variable labels
vlab <- c("maxBreadth", "basibHeight", "basialLength", "nasalHeight")
# fit manova model, classically and robustly
sk.mod <- lm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
sk.rmod <- robmlm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
# standard mlm methods apply here
coefficients(sk.rmod)
# index plot of weights
plot(sk.rmod$weights, type="h", xlab="Case Index", ylab="Robust mlm weight", col="gray")
points(sk.rmod$weights, pch=16, col=Skulls$epoch)
axis(side=1, at=15+seq(0,120,30), labels=levels(Skulls$epoch), tick=FALSE, cex.axis=1)
# heplots to see effect of robmlm vs. mlm
heplot(sk.mod, hypotheses=list(Lin="epoch.L", Quad="epoch.Q"),
    xlab=vlab[1], ylab=vlab[2], cex=1.25, lty=1)
heplot(sk.rmod, hypotheses=list(Lin="epoch.L", Quad="epoch.Q"),
    add=TRUE, error.ellipse=TRUE, lwd=c(2,2), lty=c(2,2),
    term.labels=FALSE, hyp.labels=FALSE, err.label="")
##############
# Pottery data
pottery.mod <- lm(cbind(Al,Fe,Mg,Ca,Na)~Site, data=Pottery)
pottery.rmod <- robmlm(cbind(Al,Fe,Mg,Ca,Na)~Site, data=Pottery)
Anova(pottery.mod)
Anova(pottery.rmod)
# index plot of weights
plot(pottery.rmod$weights, type="h")
points(pottery.rmod$weights, pch=16, col=Pottery$Site)
# heplots to see effect of robmlm vs. mlm
heplot(pottery.mod, cex=1.3, lty=1)
heplot(pottery.rmod, add=TRUE, error.ellipse=TRUE, lwd=c(2,2), lty=c(2,2),
    term.labels=FALSE, err.label="")
###############
# Prestige data
```

```
# treat women and prestige as response variables for this example
prestige.mod <- lm(cbind(women, prestige) ~ income + education + type, data=Prestige)
prestige.rmod <- robmlm(cbind(women, prestige) ~ income + education + type, data=Prestige)
coef(prestige.mod)
coef(prestige.rmod)
# how much do coefficients change?
round(coef(prestige.mod) - coef(prestige.rmod),3)
# pretty plot of case weights
plot(prestige.rmod$weights, type="h", xlab="Case Index", ylab="Robust mlm weight", col="gray")
points(prestige.rmod$weights, pch=16, col=Prestige$type)
legend(0, 0.7, levels(Prestige$type), pch=16, col=palette()[1:3], bg="white")
heplot(prestige.mod, cex=1.4, lty=1)
heplot(prestige.rmod, add=TRUE, error.ellipse=TRUE, lwd=c(2,2), lty=c(2,2),
    term.labels=FALSE, err.label="")
```


## Rohwer

## Rohwer Data Set

## Description

Data from an experiment by William D. Rohwer on kindergarten children designed to examine how well performance on a set of paired-associate (PA) tasks can predict performance on some measures of aptitude and achievement.

## Usage

data(Rohwer)

## Format

A data frame with 69 observations on the following 10 variables.
group a numeric vector, corresponding to SES
SES Socioeconomic status, a factor with levels Hi Lo
SAT a numeric vector: score on a Student Achievement Test
PPVT a numeric vector: score on the Peabody Picture Vocabulary Test
Raven a numeric vector: score on the Raven Progressive Matrices Test
n a numeric vector: performance on a 'named' PA task
$s$ a numeric vector: performance on a 'still' PA task
ns a numeric vector: performance on a 'named still' PA task
na a numeric vector: performance on a 'named action' PA task
ss a numeric vector: performance on a 'sentence still' PA task

## Details

The variables SAT, PPVT and Raven are responses to be potentially explained by performance on the paired-associate (PA) learning taskn, $s$, $n s$, na, and ss.

## Source

Timm, N.H. 1975). Multivariate Analysis with Applications in Education and Psychology. Wadsworth (Brooks/Cole), Examples 4.3 (p. 281), 4.7 (p. 313), 4.13 (p. 344).

## References

Friendly, M. (2007). HE plots for Multivariate General Linear Models. Journal of Computational and Graphical Statistics, 16(2) 421-444. http: //datavis.ca/papers/jcgs-heplots.pdf

## Examples

```
str(Rohwer)
## ANCOVA, assuming equal slopes
rohwer.mod <- lm(cbind(SAT, PPVT, Raven) ~ SES + n + s + ns + na + ss, data=Rohwer)
Anova(rohwer.mod)
# Visualize the ANCOVA model
heplot(rohwer.mod)
# Add ellipse to test all 5 regressors
heplot(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")))
# View all pairs
pairs(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")))
# or 3D plot
## Not run:
col <- c("red", "green3", "blue", "cyan", "magenta", "brown", "gray")
heplot3d(rohwer.mod, hypotheses=list("Regr" = c("n", "s", "ns", "na", "ss")),
            col=col, wire=FALSE)
## End(Not run)
## fit separate, independent models for Lo/Hi SES
rohwer.ses1 <- lm(cbind(SAT, PPVT, Raven) ~ n + s + ns + na + ss, data=Rohwer, subset=SES=="Hi")
rohwer.ses2 <- lm(cbind(SAT, PPVT, Raven) ~ n + s + ns + na + ss, data=Rohwer, subset=SES=="Lo")
# overlay the separate HE plots
heplot(rohwer.ses1, ylim=c(40,110),col=c("red", "black"))
heplot(rohwer.ses2, add=TRUE, col=c("blue", "black"), grand.mean=TRUE, error.ellipse=TRUE)
```


## Description

In a classic experiment carried out from 1918 to 1934, growth of apple trees of six different rootstocks were compared on four measures of size.

## Usage

data(RootStock)

## Format

A data frame with 48 observations on the following 5 variables.
rootstock a factor with levels 123456
girth4 a numeric vector: trunk girth at 4 years ( $\mathrm{mm} \times 100$ )
ext4 a numeric vector: extension growth at 4 years (m)
girth15 a numeric vector: trunk girth at 15 years ( $\mathrm{mm} \times 100$ )
weight15 a numeric vector: weight of tree above ground at 15 years ( lb x 1000 )

## Details

This is a balanced, one-way MANOVA design, with $n=8$ trees for each rootstock.

## Source

Andrews, D. and Herzberg, A. (1985). Data: A Collection of Problems from Many Fields for the Student and Research Worker Springer-Verlag, pp. 357-360.

## References

Rencher, A. C. (1995). Methods of Multivariate Analysis. New York: Wiley, Table 6.2

## Examples

```
data(RootStock)
## maybe str(RootStock) ; plot(RootStock) ...
root.mod <- lm(cbind(girth4, ext4, girth15, weight15) ~ rootstock, data=RootStock)
Anova(root.mod)
pairs(root.mod)
# test two orthogonal contrasts among the rootstocks
hyp <- matrix(c(2,-1,-1,-1,-1,2, 1, 0,0,0,0,-1), 2, 6, byrow=TRUE)
linearHypothesis(root.mod, hyp)
```

    heplot(root.mod, hypotheses=list(Contrasts=hyp, C1=hyp[1,], C2=hyp[2,]))
    heplot1d(root.mod, hypotheses=list(Contrasts=hyp, C1=hyp[1,], C2=hyp[2,]))
    
## Description

Siotani et al. (1985) describe a study of Japanese rice wine (sake) used to investigate the relationship between two subjective ratings (taste and smell) and a number of physical measurements on 30 brands of sake.
These data provide one example of a case where a multivariate regression doesn't benefit from having multiple outcome measures, using the standard tests. Barrett (2003) uses this data to illustrate influence measures for multivariate regression models.

## Usage

data(Sake)

## Format

A data frame with 30 observations on the following 10 variables.
taste mean taste rating
smell mean smell rating
pH pH measurement
acidity1 one measure of acidity
acidity2 another measure of acidity
sake Sake-meter score
rsugar direct reducing sugar content
tsugar total sugar content
alcohol alcohol content
nitrogen formol-nitrogen content

## Details

The taste and smell values are the mean ratings of 10 experts on some unknown scale.

## Source

Siotani, M. Hayakawa, T. \& Fujikoshi, Y. (1985). Modern Multivariate Statistical Analysis: A Graduate Course and Handbook. American Sciences Press, p. 217.

## References

Barrett, B. E. (2003). Understanding Influence in Multivariate Regression. Communications in Statistics - Theory and Methods 32 (3), 667-680.

## Examples

```
data(Sake)
# quick look at the data
boxplot(scale(Sake))
Sake.mod <- lm(cbind(taste,smell) ~ ., data=Sake)
library(car)
Anova(Sake.mod)
predictors <- colnames(Sake)[-(1:2)]
# overall multivariate regression test
linearHypothesis(Sake.mod, predictors)
heplot(Sake.mod, hypotheses=list("Regr" = predictors))
```

```
schooldata School Data
```


## Description

School Data, from Charnes et al. (1981). The aim is to explain scores on 3 different tests, reading, mathematics and selfesteem from 70 school sites by means of 5 explanatory variables.

## Usage

data(schooldata)

## Format

A data frame with 70 observations on the following 8 variables.
education education level of mother as measured in terms of percentage of high school graduates among female parents
occupation highest occupation of a family member according to a pre-arranged rating scale visit parental visits index representing the number of visits to the school site counseling parent counseling index calculated from data on time spent with child on schoolrelated topics such as reading together, etc.
teacher number of teachers at a given site
reading total reading score as measured by the Metropolitan Achievement Test mathematics total mathematics score as measured by the Metropolitan Achievement Test selfesteem Coopersmith Self-Esteem Inventory, intended as a measure of self-esteem

## Details

This dataset was shamelessly borrowed from the FRB package.
The relationships among these variables are unusual, a fact only revealed by plotting.

## Source

A. Charnes, W.W. Cooper and E. Rhodes (1981). Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through. Management Science, 27, 668-697.

## Examples

```
data(schooldata)
# initial screening
plot(schooldata)
# better plot
library(corrgram)
corrgram(schooldata, lower.panel=panel.ellipse, upper.panel=panel.pts)
#fit the MMreg model
school.mod <- lm(cbind(reading, mathematics, selfesteem) ~
education + occupation + visit + counseling + teacher, data=schooldata)
# shorthand
school.mod <- lm(cbind(reading, mathematics, selfesteem) ~ ., data=schooldata)
Anova(school.mod)
heplot(school.mod)
heplot3d(school.mod)
# robust model, using robmlm()
school.rmod <- robmlm(cbind(reading, mathematics, selfesteem) ~ ., data=schooldata)
# note that counseling is now significant
Anova(school.rmod)
# compare classical HEplot with robust
heplot(school.mod, cex=1.4, lty=1, fill=TRUE, fill.alpha=0.1)
heplot(school.rmod, add=TRUE, error.ellipse=TRUE, lwd=c(2,2), lty=c(2,2),
term.labels=FALSE, err.label="", fill=TRUE)
```

Skulls Egyptian Skulls

## Description

Measurements made on Egyptian skulls from five epochs.

## Usage

```
data("Skulls")
```


## Format

A data frame with 150 observations on the following 5 variables.
epoch the epoch the skull as assigned to, an ordered factor with levels c4000BC c3300BC, c1850BC, c200BC, and CAD150, where the years are only given approximately, of course.
mb maximal breadth of the skull.
bh basibregmatic height of the skull.
bl basialiveolar length of the skull.
nh nasal height of the skull.

## Details

The epochs correspond to the following periods of Egyptian history: 1 - the early predynastic period (circa 4000 BC ); 2 - the late predynastic period (circa 3300 BC ); 3 - the 12th and 13th dynasties (circa 1850 BC ); 4 - the Ptolemiac period (circa 200 BC ); 5 - the Roman period (circa 150 AD).
The question is whether the measurements change over time. Non-constant measurements of the skulls over time would indicate interbreeding with immigrant populations.
Note that using polynomial contrasts for epoch essentially treats the time points as equally spaced.

## Source

D. J. Hand, F. Daly, A. D. Lunn, K. J. McConway and E. Ostrowski (1994). A Handbook of Small Datasets, Chapman and Hall/CRC, London.

## References

Thomson, A. and Randall-Maciver, R. (1905) Ancient Races of the Thebaid, Oxford: Oxford University Press.
Hand, D. J., F. Daly, A. D. Lunn, K. J. McConway and E. Ostrowski (1994). A Handbook of Small Datasets, Chapman and Hall/CRC, London.

## Examples

```
data(Skulls)
# make shorter labels for epochs
Skulls$epoch <- factor(Skulls$epoch, labels=sub("c","",levels(Skulls$epoch)))
# variable labels
vlab <- c("maxBreadth", "basibHeight", "basialLength", "nasalHeight")
# fit manova model
sk.mod <- lm(cbind(mb, bh, bl, nh) ~ epoch, data=Skulls)
Manova(sk.mod)
summary(Manova(sk.mod))
```

```
# test trends over epochs
linearHypothesis(sk.mod, "epoch.L") # linear component
linearHypothesis(sk.mod, "epoch.Q") # quadratic component
# typical scatterplots are not very informative
scatterplot(mb ~ bh|epoch, data=Skulls,
    ellipse=TRUE, levels=0.68, smooth=FALSE, legend.coords="topright")
scatterplot(mb ~ bl|epoch, data=Skulls,
    ellipse=TRUE, levels=0.68, smooth=FALSE, legend.coords="topright")
# HE plots
heplot(sk.mod, hypotheses=list(Lin="epoch.L", Quad="epoch.Q"), xlab=vlab[1], ylab=vlab[2])
pairs(sk.mod, hypotheses=list(Lin="epoch.L", Quad="epoch.Q"), var.labels=vlab)
# 3D plot shows that nearly all of hypothesis variation is linear!
## Not run:
heplot3d(sk.mod, hypotheses=list(Lin="epoch.L", Quad="epoch.Q"), col=c("pink", "blue"))
# view in canonical space
if (require(candisc)) {
sk.can <- candisc(sk.mod)
sk.can
heplot(sk.can)
heplot3d(sk.can)
}
## End(Not run)
```

SocGrades Grades in a Sociology Course

## Description

The data set SocGrades contains four outcome measures on student performance in an introductory sociology course together with six potential predictors. These data were used by Marascuilo and Levin (1983) for an example of canonical correlation analysis, but are also suitable as examples of multivariate multiple regression, MANOVA, MANCOVA and step-down analysis in multivariate linear models.

## Usage

data(SocGrades)

## Format

A data frame with 40 observations on the following 10 variables.
class Social class, an ordered factor with levels $1>2>3$
sex sex, a factor with levels F M
gpa grade point average
boards College Board test scores
hssoc previous high school unit in sociology, a factor with 2 no, yes
pretest score on course pretest
midterm1 score on first midterm exam
midterm2 score on second midterm exam
final score on final exam
eval course evaluation

## Details

midterm1, midterm2, final, and possibly eval are the response variables. All other variables are potential predictors.

The factors class, sex, and hssoc can be used with as. numeric in correlational analyses.

## Source

Marascuilo, L. A. and Levin, J. R. (1983). Multivariate Statistics in the Social Sciences Monterey, CA: Brooks/Cole, Table 5-1, p. 192.

## Examples

```
data(SocGrades)
# basic MLM
grades.mod <- lm(cbind(midterm1, midterm2, final, eval) ~
class + sex + gpa + boards + hssoc + pretest, data=SocGrades)
Anova(grades.mod, test="Roy")
clr <- c("red", "blue", "darkgreen", "magenta", "brown", "black", "darkgray")
heplot(grades.mod, col=clr)
pairs(grades.mod, col=clr)
## Not run:
heplot3d(grades.mod, col=clr, wire=FALSE)
## End(Not run)
if (require(candisc)) {
# calculate canonical results for all terms
grades.can <- candiscList(grades.mod)
# extract canonical R^2s
unlist(lapply(grades.can, function(x) x$canrsq))
# plot class effect in canonical space
heplot(grades.can, term="class", scale=4)
```

```
# 1 df terms: show canonical scores and weights for responses
plot(grades.can, term="sex")
plot(grades.can, term="gpa")
plot(grades.can, term="boards")
}
```

SocialCog Social Cognitive Measures in Psychiatric Groups

## Description

The general purpose of the study (Hartman, 2016, Heinrichs et al. (2015)) was to evaluate patterns and levels of performance on neurocognitive measures among individuals with schizophrenia and schizoaffective disorder using a well-validated, comprehensive neurocognitive battery specifically designed for individuals with psychosis (Heinrichs et al. (2008))
The data here are for a subset of the observations in NeuroCog for which measures on various scales of social cognition were also available. Interest here is on whether the schizophrenia group can be distinguished from the schizoaffective group on these measures.
The Social Cognitive measures were designed to tap various aspects of the perception and cognitive procession of emotions of others. Emotion perception was assessed using a Managing Emotions (MgeEmotions) score from the MCCB. A "theory of mind" (ToM) score assessed ability to read the emotions of others from photographs of the eye region of male and female faces. Two other measures, externalizing bias (ExtBias) and personalizing bias (PersBias) were calculated from a scale measuring the degree to which individuals attribute internal, personal or situational causal attributions to positive and negative social events.

## Usage

```
data("SocialCog")
```


## Format

A data frame with 139 observations on the following 5 variables.
Dx Diagnostic group, a factor with levels Schizophrenia, Schizoaffective, Control
MgeEmotions Score on the Managing emotions test, a numeric vector
ToM Score on the The Reading the Mind in the Eyes test (theory of mind), a numeric vector
ExtBias Externalizing Bias score, a numeric vector
PersBias Personal Bias score, a numeric vector

## Details

See NeuroCog for a description of the sample. Only those with complete data on all the social cognitive measures are included in this data set.

There is one extreme outlier in the schizophrenia group and other possible outliers in the control group, left in here for tutorial purposes.

## Source

Hartman, L. I. (2016). Schizophrenia and Schizoaffective Disorder: One Condition or Two? Unpublished PhD dissertation, York University.
Heinrichs, R.W., Pinnock, F., Muharib, E., Hartman, L.I., Goldberg, J.O., \& McDermid Vaz, S. (2015). Neurocognitive normality in schizophrenia revisited. Schizophrenia Research: Cognition, 2 (4), 227-232. doi: 10.1016/j.scog.2015.09.001

## Examples

```
data(SocialCog)
SC.mod <- lm(cbind(MgeEmotions, ToM, ExtBias, PersBias) ~ Dx, data=SocialCog)
SC.mod
Anova(SC.mod)
# test hypotheses of interest in terms of contrasts
print(linearHypothesis(SC.mod, "Dx1"), SSP=FALSE)
print(linearHypothesis(SC.mod, "Dx2"), SSP=FALSE)
#' ## HE plots
heplot(SC.mod, hypotheses=list("Dx1"="Dx1", "Dx2"="Dx2"),
fill=TRUE, fill.alpha=.1)
pairs(SC.mod, fill=c(TRUE,FALSE), fill.alpha=.1)
```

statList

Calculate statistics for levels of factors

## Description

statList provides a general method for calculating univariate or multivariate statistics for a matrix or data.frame stratified by one or more factors.

## Usage

statList(X, factors, FUN, drop = FALSE, ...)
colMeansList(X, factors, drop = FALSE, ...)
covList(X, factors, drop = FALSE, ...)

## Arguments

$X \quad$ A matrix or data frame containing the variables to be summarized
factors A vector, matrix or data frame containing the factors for which $X$ is to be summarized. If factors is not specified, the result is calculated for all of the data in X.

| FUN | A function to be applied to the pieces of $X$, as split by factors. |
| :--- | :--- |
| drop | Logical, indicating whether empty levels of factors are to be dropped from the <br> result. |
| $\ldots$ | Other arguments, passed to FUN. |

## Details

statList is the general function. X is first split by factors, and FUN is applied to the result. colMeansList and covList are just calls to statList with the appropriate FUN.

## Value

Returns a list of items corresponding to the unique elements in factors, or the interaction of factors. Each item is the result of applying FUN to that collection of rows of $X$. The items are named according to the levels in factors.

## Author(s)

Michael Friendly

## See Also

colMeans, termMeans

## Examples

```
# grand means
statList(iris[,1:4], FUN=colMeans)
# species means
statList(iris[,1:4], iris$Species, FUN=colMeans)
# same
colMeansList(iris[,1:4], iris$Species)
# var-cov matrices, by species
covList(iris[,1:4], iris$Species)
# multiple factors
iris$Dummy <- sample(c("Hi","Lo"),150, replace=TRUE)
colMeansList(iris[,1:4], iris[,5:6])
```


## Description

termMeans is a utility function designed to calculate means for the levels of factor(s) for any term in a multivariate linear model.

## Usage

termMeans(mod, term, label.factors=FALSE, abbrev.levels=FALSE)

## Arguments

mod An mlm model object
term A character string indicating a given term in the model. All factors in the term must be included in the model, even if they are in the model data frame.
label.factors If true, the rownames for each row in the result include the name(s) of the factor(s) involved, followed by the level values. Otherwise, the rownames include only the levels of the factor(s), with multiple factors separated by ' $:$ '
abbrev.levels Either a logical or an integer, specifying whether the levels values of the factors in the term are to be abbreviated in constructing the rownames. An integer specifies the minimum length of the abbreviation for each factor in the term.

## Value

Returns a matrix whose columns correspond to the response variables in the model and whose rows correspond to the levels of the factor(s) in the term.

## Author(s)

Michael Friendly

## See Also

aggregate, colMeans
statList, colMeansList

## Examples

```
factors <- expand.grid(A=factor(1:3),B=factor(1:2),C=factor(1:2))
n <- nrow(factors)
responses <-data.frame(Y1=10+round(10*rnorm(n)),Y2=10+round(10*rnorm(n)))
test <- data.frame(factors, responses)
mod <- lm(cbind(Y1,Y2) ~ A*B, data=test)
termMeans(mod, "A")
```

```
termMeans(mod, "A:B")
termMeans(mod, "A:B", label.factors=TRUE)
## Not run:
termMeans(mod, "A:B:C") # generates an error
## End(Not run)
plastic.mod <- lm(cbind(tear, gloss, opacity) ~ rate*additive, data=Plastic)
colors = c("red", "darkblue", "darkgreen", "brown")
heplot(plastic.mod, col=colors, cex=1.25)
# add means for interaction term
intMeans <- termMeans(plastic.mod, 'rate:additive', abbrev=2)
points(intMeans[,1], intMeans[,2], pch=18, cex=1.2, col="brown")
text(intMeans[,1], intMeans[,2], rownames(intMeans), adj=c(0.5,1), col="brown")
```

trans.colors Make Colors Transparent

## Description

Takes a vector of colors (as color names or rgb hex values) and adds a specified alpha transparency to each.

## Usage

trans.colors(col, alpha $=0.5$, names $=$ NULL)

## Arguments

col A character vector of colors, either as color names or rgb hex values
alpha alpha transparency value(s) to apply to each color (0 means fully transparent and 1 means opaque)
names optional character vector of names for the colors

## Details

Colors (col) and alpha need not be of the same length. The shorter one is replicated to make them of the same length.

## Value

A vector of color values of the form "\#rrggbbaa"

## Author(s)

Michael Friendly

## See Also

col2rgb, rgb, adjustcolor,

## Examples

```
trans.colors(palette(), alpha=0.5)
# alpha can be vectorized
trans.colors(palette(), alpha=seq(0, 1, length=length(palette())))
# lengths need not match: shorter one is repeated as necessary
trans.colors(palette(), alpha=c(.1, .2))
trans.colors(colors()[1:20])
# single color, with various alphas
trans.colors("red", alpha=seq(0,1, length=5))
# assign names
trans.colors("red", alpha=seq(0,1, length=5), names=paste("red", 1:5, sep=""))
```

VocabGrowth Vocabulary growth data

## Description

Data from the Laboratory School of the University of Chicago. They consist of scores from a cohort of pupils in grades $8-11$ on the vocabulary section of the Cooperative Reading Test. The scores are scaled to a common, but arbitrary origin and unit of measurement, so as to be comparable over the four grades.

## Usage

data(VocabGrowth)

## Format

A data frame with 64 observations on the following 4 variables.
grade8 Grade 8 vocabulary score
grade9 Grade 9 vocabulary score
grade10 Grade 10 vocabulary score
grade11 Grade 11 vocabulary score

## Details

Since these data cover an age range in which physical growth is beginning to decelerate, it is of interest whether a similar effect occurs in the acquisition of new vocabulary.

## Source

R.D. Bock, Multivariate statistical methods in behavioral research, McGraw-Hill, New York, 1975, pp453.

## References

Friendly, Michael (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04.
Keesling, J.W., Bock, R.D. et al, "The Laboratory School study of vocabulary growth", University of Chicago, 1975.

## Examples

```
data(VocabGrowth)
# Standard Multivariate & Univariate repeated measures analysis
Vocab.mod <- lm(cbind(grade8,grade9,grade10,grade11) ~ 1, data=VocabGrowth)
idata <-data.frame(grade=ordered(8:11))
Anova(Vocab.mod, idata=idata, idesign=~grade, type="III")
##Type III Repeated Measures MANOVA Tests: Pillai test statistic
## Df test stat approx F num Df den Df Pr(>F)
##(Intercept) 1 0.653 118.498 1 63 4.115e-16 ***
##grade 1 0.826 96.376 3 61 < 2.2e-16 ***
heplot(Vocab.mod, type="III", idata=idata, idesign=~grade, iterm="grade",
main="HE plot for Grade effect")
### doing this 'manually' by explicitly transforming Y -> Y M
# calculate Y M, using polynomial contrasts
trends <- as.matrix(VocabGrowth) %*% poly(8:11, degree=3)
colnames(trends)<- c("Linear", "Quad", "Cubic")
# test all trend means = 0 == Grade effect
within.mod <- lm(trends ~ 1)
Manova(within.mod)
heplot(within.mod, terms="(Intercept)", col=c("red", "blue"), type="3",
    term.labels="Grade",
    main="HE plot for Grade effect")
mark.H0()
```


## Description

Contrived data on weight loss and self esteem over three months, for three groups of individuals: Control, Diet and Diet + Exercise. The data constitute a double-multivariate design.

## Usage

data(WeightLoss)

## Format

A data frame with 34 observations on the following 7 variables.
group a factor with levels Control Diet DietEx.
wl1 Weight loss at 1 month
wl2 Weight loss at 2 months
wl3 Weight loss at 3 months
se1 Self esteem at 1 month
se2 Self esteem at 2 months
se3 Self esteem at 3 months

## Details

Helmert contrasts are assigned to group, comparing Control vs. (Diet DietEx) and Diet vs. DietEx.

## Source

Originally taken from http: //www.csun.edu/~ata20315/psy524/main.htm, but modified slightly

## References

Friendly, Michael (2010). HE Plots for Repeated Measures Designs. Journal of Statistical Software, 37(4), 1-40.
doi: 10.18637/jss.v037.i04.

## Examples

```
data(WeightLoss)
str(WeightLoss)
table(WeightLoss$group)
contrasts(WeightLoss$group) <- matrix(c(-2,1,1, 0, -1, 1),ncol=2)
(wl.mod<-lm(cbind(wl1,wl2,wl3,se1,se2,se3)~group, data=WeightLoss))
heplot(wl.mod, hypotheses=c("group1", "group2"))
pairs(wl.mod, variables=1:3)
pairs(wl.mod, variables=4:6)
```

```
# within-S variables
within <- data.frame(measure=rep(c("Weight loss", "Self esteem"), each=3), month=rep(ordered(1:3),2))
# doubly-multivariate analysis: requires car 2.0+
## Not run:
if (packageDescription("car")[["Version"]] >= 2) {
imatrix <- matrix(c(
1,0,-1, 1, 0, 0,
1,0, 0,-2, 0, 0,
1,0, 1, 1, 0, 0,
0,1, 0, 0,-1, 1,
0,1, 0, 0, 0,-2,
0,1, 0, 0, 1, 1), 6, 6, byrow=TRUE)
# NB: for heplots the columns of imatrix should have names
colnames(imatrix) <- c("WL", "SE", "WL.L", "WL.Q", "SE.L", "SE.Q")
rownames(imatrix) <- colnames(WeightLoss)[-1]
(imatrix <- list(measure=imatrix[,1:2], month=imatrix[,3:6]))
contrasts(WeightLoss$group) <- matrix(c(-2,1,1, 0,-1,1), ncol=2)
(wl.mod<-lm(cbind(wl1, wl2, wl3, se1, se2, se3)~group, data=WeightLoss))
(wl.aov <- Anova(wl.mod, imatrix=imatrix, test="Roy"))
heplot(wl.mod, imatrix=imatrix, iterm="group:measure")
}
## End(Not run)
# do the correct analysis 'manually'
unit <- function(n, prefix="") {
J <-matrix(rep(1, n), ncol=1)
rownames(J) <- paste(prefix, 1:n, sep="")
J
}
measure <- kronecker(diag(2), unit(3, 'M')/3, make.dimnames=TRUE)
colnames(measure)<- c('WL', 'SE')
between <- as.matrix(WeightLoss[,-1])
between.mod <- lm(between ~ group, data=WeightLoss)
Anova(between.mod)
heplot(between.mod, hypotheses=c("group1", "group2"),
xlab="Weight Loss", ylab="Self Esteem",
col=c("red", "blue", "brown"),
main="Weight Loss & Self Esteem: Group Effect")
month <- kronecker(diag(2), poly(1:3), make.dimnames=TRUE)
colnames(month)<- c('WL', 'SE')
trends <- as.matrix(WeightLoss[,-1])
within.mod <- lm(trends ~ group, data=WeightLoss)
Anova(within.mod)
heplot(within.mod)
```

```
heplot(within.mod, hypotheses=c("group1", "group2"),
xlab="Weight Loss", ylab="Self Esteem",
type="III", remove.intercept=FALSE,
term.labels=c("month", "group:month"),
main="Weight Loss & Self Esteem: Within-S Effects")
mark.H0()
```


## Index

```
* aplot
    arrow3d, 8
    cross3d, 24
    ellipse3d.axes, 27
    heplot,35
    heplot1d, 41
    heplot3d,44
    heplots-package, 3
    mark.H0,59
* color
    trans.colors,98
* datasets
    AddHealth,5
    Adopted,6
    Bees,10
    Diabetes, 25
    FootHead, 30
    Headache, 33
    Hernior,48
    mathscore, }6
    MockJury, }6
    NeuroCog, 62
    NLSY, }6
    Oslo,66
    Parenting,69
    Plastic,71
    Pottery2,75
    Probe,77
    RatWeight,79
    ReactTime, 80
    Rohwer, }8
    RootStock, }8
    Sake, }8
    schooldata, }8
    Skulls,90
    SocGrades,92
    SocialCog, 94
    VocabGrowth, }9
    WeightLoss, 100
```

```
* dynamic
    cross3d, 24
    ellipse3d.axes,27
    heplot3d,44
* effect size
    etasq, 28
* hgraph
    plot.boxM, 72
* hplot
    coefplot,14
    covEllipses,18
    cqplot,21
    heplot, 35
    heplot1d,41
    heplot3d, 44
    heplots-package, 3
    interpPlot,50
    pairs.mlm,67
    plot.robmlm, 73
* htest
    bartlettTests,9
        leveneTests,55
* manip
        colDevs,16
        gsorth, 31
        logdetCI, 56
        termMeans,97
* models
        termMeans,97
* multivariate
    etasq, 28
    heplot, 35
    heplot1d,41
    heplot3d,44
    heplots-package, 3
    Mahalanobis,58
    pairs.mlm, 67
    robmlm,81
    statList,95
```

```
* package
    heplots-package, 3
* robust
        robmlm, 81
* strength of association
        etasq, 28
* utilities
        statList, }9
AddHealth, 5
adjustcolor,99
Adopted,6
aggregate, }9
animation,51
Anova, 4, 5, 29, 36, 39, 42, 43, 45, 47, 68, 82
anova, 31
arrow3d, 8
bartlett.test, }
bartlettTests, 9,56
Bees,10
bg,46
boxM, 9, 12, 20, 56, 57, 72, 73
candisc, 38, 39
coefficients, }8
coefplot,14
coefplot.mlm, 39
col2rgb, }9
colDevs,16
colMeans, 17, 96, 97
colMeansList,97
colMeansList(statList), }9
confidenceEllipse, 15, 16
cov.rob, 18, 20,58
cov.trob, 83, }8
covEllipses, 14, 18
covList(statList), 95
cqplot,21
cross3d, 24, 59
dataEllipse,51
Diabetes, }2
dotchart, }7
effects,83
ellipse3d, 28
ellipse3d.axes,27
etasq, 28
```

ex1605, 6, 7
fitted.values, 83
FootHead, 30
gsorth, 31
Headache, 33
heplot, $3,20,35,43,47,54,68,69,82$
heplot.candisc, 38, 39
heplot1d, 4, 39, 41
heplot3d, 3, 39, 43, 44, 69
heplot3d.candisc, 38, 47
heplots (heplots-package), 3
heplots-package, 3
Hernior, 48
interpPlot, 50
label.ellipse, 15, 36, 39, 53
leveneTest, 14, 55, 56
leveneTests, 55
linearHypothesis, 4, 5, 36, 38, 39, 41, 43, 45, 47
lines3d, 8
lm, 82, 83
logdetCI, 56, 73
Mahalanobis, 23, 58
mahalanobis, 58
manova, 5
mark.H0, 37, 39, 59
material3d, 8
mathscore, 60
MockJury, 61
movie3d, 47
na.omit, 82
NeuroCog, 62, 94
NLSY, 64
options, 82
Oslo, 66
OsloTransect, 66, 67
pairs, 68
pairs.mlm, 3, 39, 43, 47, 67
palette, 22
par, 22
Parenting, 69

```
Plastic,71
play3d,47
plot,74
plot.boxM, 14, 56, 57, 72
plot.robmlm, }7
points3d,59
Pottery, 75, 76
Pottery2,75
print.robmlm(robmlm), 81
print.summary.robmlm(robmlm), 81
Probe, }7
Probe1 (Probe), 77
Probe2 (Probe), 77
psi.bisquare, 83
qqPlot,23
qqplot, 23
qqtest,23
qr, 32
RatWeight,79
ReactTime, }8
residuals, 83
rgb,99
rlm, 83, 84
robmlm, 73, 74, }8
Rohwer, 85
RootStock, }8
Sake, }8
schooldata, 89
segments3d, 8, 25, 28
shade3d,46
showLabels, 22, 50, 51
Skulls,90
SocGrades,92
SocialCog, }9
statList, 95,97
summary.boxM (boxM), 12
summary.robmlm (robmlm), 81
sweep, 16,17
termMeans, 96, 97
text3d,28
trans.colors, 39, 98
vcov, }8
viewpoint,46
VocabGrowth, }9
```

