

# Package ‘mmeln’

December 18, 2019

**Type** Package

**Title** Estimation of Multinormal Mixture Distribution

**Version** 1.4

**Date** 2019-12-17

**Author** Charles-Edouard Giguere

**Maintainer** Charles-Edouard Giguere <ce.giguere@gmail.com>

**Description** Fit multivariate mixture of normal distribution using covariance structure.

**License** GPL-3

**LazyLoad** yes

**Repository** CRAN

**Encoding** UTF-8

**NeedsCompilation** no

**Date/Publication** 2019-12-17 23:00:02 UTC

## R topics documented:

mmeln-package . . . . .	2
dmnorm . . . . .	3
estim . . . . .	4
exY . . . . .	6
mmeln . . . . .	6
plot.mmeln,logLik.mmeln,anova.mmeln,print.mmeln . . . . .	8
post.mmeln,entropy.mmeln . . . . .	10
<b>Index</b>	<b>12</b>

mmeln-package

*Estimation of Multinormal Mixture Distribution***Description**

Fit multivariate mixture of normal distribution using covariance structure.

**Details**

The DESCRIPTION file:

```
Package:          mmeln
Type:            Package
Title:           Estimation of Multinormal Mixture Distribution
Version:         1.4
Date:            2019-12-17
Author:          Charles-Edouard Giguere
Maintainer:      Charles-Edouard Giguere <ce.giguere@gmail.com>
Description:     Fit multivariate mixture of normal distribution using covariance structure.
License:         GPL-3
LazyLoad:        yes
Repository:      CRAN
Encoding:        UTF-8
NeedsCompilation: no
```

Index of help topics:

```
dmnorm          Multivariate Normal Density Function
estim           Maximum Likelihood estimation of the model
                parameters
exY             A two mixture example
mmeln           mmeln : mixture of multivariate normal
mmeln.package   Estimation of Multinormal Mixture Distribution
plot.mmeln      Utility methods for objects of class mmeln
post.mmeln      Posterior probabilities, entropy for mmeln
                object
```

~~ An overview of how to use the package, including the most important ~~ ~~ functions ~~

**Author(s)**

Charles-Edouard Giguere

Maintainer: Charles-Edouard Giguere <ce.giguere@gmail.com>

**See Also**

mmeln,estim.mmeln,anova.mmeln

**Examples**

```
### load an example.
data(exY)

### estimation of the parameters of the mixture.

temps <- factor(1:3)
mmeln1 <- mmeln(Y, G = 2, form.loc = ~temps-1, form.mel = ~1, cov = "CS")
mix1 <- estim(mmeln1, mu = list(rep(1,3), rep(2,3)), tau = c(0),
             sigma = list(c(1,.6), c(1,.6)), iterlim = 100, tol = 1e-6)

mix1
anova(mix1)
plot(mix1, main="Mixture of multivariate normal")
```

dmnorm

*Multivariate Normal Density Function***Description**

Function to estimate Multivariate Normal Density Function

**Usage**

```
dmnorm(X, Mu, Sigma)
```

**Arguments**

X	A matrix or a vector (if you have only one multivariate observation) containing the data. This matrix may contain missing data.
Mu	A mean vector or a matrix where the number of column is p. If Mu is a matrix and X a vector, the density is evaluated for each value of Mu specified in the matrix Mu
Sigma	The covariance matrix. This matrix must be symmetric positive definite (all eigen values are positive. see <a href="#">eigen</a> )

**Details**

This methods compute the value of the density function for a given data and a given set of parameters. It works like the R command [dnorm](#) in the stats package. Although this methods can be used directly it is not intended this way. If you want to estimate density of multivariate normal distribution, the library [mvtnorm](#) is more appropriate

**Value**

This command return a vector of density.

**Note**

This function can be used as a standalone but is implemented here for use within the `mmeln` package

**Author(s)**

Charles-Édouard Giguère

**References**

M.S. Srivastava (2002), *Methods of Multivariate Statistics*, WILEY

**See Also**

`mmeln`, `eigen`

**Examples**

```
dmnorm(1:3, 1:3, diag(3))
```

---

estim

*Maximum Likelihood estimation of the model parameters*

---

**Description**

Compute the MLE of the model parameters using the E-M (Expectation-Maximization) algorithm

**Usage**

```
## S3 method for class 'mmeln'
estim(X, ..., mu=NULL, tau=NULL, sigma=NULL, random.start=FALSE, iterlim=500, tol=1e-8)
```

**Arguments**

<code>X</code>	An object of type <code>mmeln</code> containing the design of the model, see <a href="#">mmeln</a>
<code>...</code>	For the moments no other arguments can be added
<code>mu</code>	A list of length <code>X\$G</code> containing the starting value for the location parameters
<code>tau</code>	The starting value for the mixture parameters
<code>sigma</code>	A list of length <code>X\$G</code> containing the starting value for the covariances parameters
<code>random.start</code>	A True/False value indicating if the starting parameters should be given at random. If true the starting values are not needed.
<code>iterlim</code>	The maximum number of iterations allowed
<code>tol</code>	Tolerance, degree of precision required to stop the iterative process

**Details**

Methods estim.mmeln... are used by the estim function but are of no use outside this method.

**Value**

Retourne un objet de type "mmeln" & "mmelnSOL" les arguments suivants :

obj\$Y	The data matrix
obj\$G	The number of groups
obj\$p	Number of column in Y
obj\$N	Number of row in Y
obj\$Xg	The list of location design matrices
obj\$pl	The number of location parameters
obj\$Z	Mixture design matrix
obj\$pm	The number of mixture parameters
obj\$cov	Covariance type
obj\$equalcov	logical value indicating if covariance is equal across group
obj\$pc	The number of covariance parameters

**Author(s)**

Charles-Édouard Giguère

**References**

- McLachlan, G. & Peel, D. (2000), Finite mixture models, Wiley
- Flury, B. D. (1997), A first course in multivariate statistics, Springer
- Pinheiro J. C. and Bates D. M. (2000), Mixed-Effects Models in S and S-PLUS, Springer
- Srivastava, M.S. (2002), Methods of Multivariate Statistics, WILEY
- Lindstrom M. J. and Bates D. M. (1988), Newton-Raphson and EM Algorithms for Linear Mixed-Effects Models for Repeated-MeasuresData, Journal of the American Statistical Association, American Statistical Association, V. 83, I. 404, P. 1014-1022

**See Also**

[mmeln.package](#)

**Examples**

```
data(exY)
### estimation of the parameters of the mixture
temps=0:2
mmeln1=mmeln(Y, G = 3, form.loc = list(~temps, ~temps + I(temps^2),
                                     ~temps + I(temps^2)), form.mel = ~SEXE, cov = "CS")
mmelnSOL1=estim(mmeln1, mu = list(c(1,1), c(2,0,0), c(3,0,0)),
                tau = c(0,0,0,0), sigma = list(c(1,0), c(1,0), c(1,0)))
```

---

exY *A two mixture example*

---

### Description

A simulated dataset used for example

### Format

Two variables are available:

**SEXE** A variable identifying sex of participants.

**Y** A three column matrix containing the data.

### Details

Half of the row follow the distribution  $N[(2,3,4)', \text{matrix}(c(1,.6,.5,.6,1,.3,.5,.3,1),3,3))]$ , the other half follow the distribution  $N[(-1,5,-2)', \text{matrix}(c(1,.6,.5,.6,1,.3,.5,.3,1),3,3))]$

---

mmeln *mmeln : mixture of multivariate normal*

---

### Description

constructor for objects of class mmeln: mixture of multivariate normal

### Usage

```
mmeln(Y, G=2, p=dim(Y)[2], form.loc=NULL, X=NULL,
form.me1=NULL, Z=NULL, cov="IND", equalcov=FALSE, param=NULL)
```

### Arguments

Y	A matrix containing the data used for estimation. This matrix may contains NA but it needs at least one observation per row. It's assumed that the missing mechanism is not related to the data under study (MAR: Missing At Random).
G	The number of groups in the mixture
p	Doesn't need to be specified. It's the dimension of the multivariate data (number of column in Y)
form.loc, X	Location design of the model. By default, the mean model is used where we estimate p mean in each group. Only one of these two parameters must be specified depending if the model is specified through a formula (See R documentation) or a design matrix. If you want to specify a different design for each group you must pass the arguments as a list. See examples below for further details. If a formula is used it must use variable of length p representing the design across time, for example : ~temps where temps=factor(1:4). If a design matrix is used, it must be of dimension p*k where k<=p

form.me1,Z	Mixture design of the model. Only one of these two parameters must be specified. The design is constant across groups. This is equivalent to multinomial regression
cov	Covariance type (for now only the CS structure is implemented). Enter either the type of covariance as a string or as numeric corresponding to the position in the following choices : 1)UN (general unstructured covariance),2 CS (Compound Symmetry with constant variance) ,3) UCS (Compound Symmetry with unconstant variance) ,4) AR1 (Auto-regressive of order 1 with constant variance), 5) UAR1 (Auto-regressive of order 1 with unconstant variance),6) IND: (diagonal structure with constant variance), 7) UIND (diagonal structure with unconstant variance)
equalcov	Logical value T/F indicating if the variance is equal across groups. Default to FALSE.
param	list of list of parameters. Usually not specified. The parameters should be estimated through the <code>estim.mmeln</code> function. param will look like this list( $\mu = \text{list}(\mu_1, \mu_2, \dots, \mu_g)$ , $\tau = \text{c}(\tau_1, \dots, \tau_k)$ , $\sigma = \text{list}(\sigma_1, \sigma_2, \dots, \sigma_g)$ ) where $\mu_i$ is the vector of location parameter in the group $i$ and $\sigma_i$ is the vector of location parameter in the group $i$ for which the length must equal the number of column in the design matrix. Also $\sigma_i$ is the vector of covariance parameters in the group $i$ . Each covariance is parameterized in a vector containing first the distinct value of standard deviation and then the distinct value of correlation from top to bottom and left to right.

## Details

This object describes the way the mixture is design and permits a lot of different modelisation of the data. Many specific methods are associated with this class of objects: `print`, `anova`, `logLik`, `post`. Once a solution is find through the `estim.mmeln` function, the object is promoted to an object of class `mmelnSOL` but inherits of all the attributes and function of the `mmeln` class but gains is own `print` method. The attributes in a `mmeln` object should be accessed via adequate function inside the `mmeln` library except if handle by an advanced user.

## Value

Retourne un objet de type "mmeln" ayant les arguments suivants :

obj\$Y	The data matrix
obj\$Y1	A list of length N containing the data in each row without the NA value.
obj\$Yv	A list of length N indicating the column where there is valid data
obj\$G	The number of groups
obj\$p	Number of column in Y
obj\$pi	A vector where $\pi[i]$ is the number of observation in row $i$
obj\$N	Number of row in Y
obj\$M	Number of total observations $\sum_{i=1}^N \pi_i$
obj\$Xg	The list of location design matrices
obj\$pl	The number of location parameters

obj\$Z	Mixture design matrix
obj\$pm	The number of mixture parameters
obj\$cov	Covariance type
obj\$equalcov	logical value indicating if covariance is equal across group
obj\$pc	The number of covariance parameters

**Author(s)**

Charles-Édouard Giguère

**References**

- McLachlan, G. & Peel, D. (2000), Finite mixture models, Wiley
- Bernard D. Flury (1997), A first course in multivariate statistics, Springer
- Pinheiro José C. & Bates Douglas M. (2000), Mixed-Effects Models in S and S-PLUS, Springer
- M.S. Srivastava (2002), Methods of Multivariate Statistics, WILEY

**See Also**

[estim.mmeln](#)

**Examples**

```
data(exY)
### estimation of the parameters of the mixture
temps <- 0:2
mmeln1 <- mmeln(Y, G = 3,
               form.loc = list(~temps, ~temps + I(temps^2), ~temps + I(temps^2)),
               form.mel = ~SEXE, cov = "CS")
```

---

`plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln`

*Utility methods for objects of class mmeln*

---

**Description**

Methods to plot, compare and assessed the log(Likelihood) of objects of class mmeln. The method cov.tsf which convert a vector of covariance parameter into a covariance matrix and multnm which performs an estimation of multinomial model are internal methods that should not be used unless by experimented user



**Usage**

```
## S3 method for class 'mmeln'
plot(x, ..., main="", xlab="Temps", ylab="Y", col=1:x$G, leg=TRUE)
## S3 method for class 'mmeln'
logLik(object, ..., param=NULL)
## S3 method for class 'mmeln'
anova(object, ..., test = TRUE)
## S3 method for class 'mmelnSOL'
print(x, ..., se.estim="MLR")
cov.tsf(param, type, p)
```

**Arguments**

x	An object of type mmeln or mmelnSOL (mmelnSOL required for the command print)
object	An object of type mmeln
main	Title of the graphic
xlab	Label of the X axis
ylab	label of the Y axis
col	Colour of the lines plotted in each group
leg	Logical value indicating if the legend is plotted or not
...	other object of type mmeln to compare (use is only valid in the anova command)
test	logical value indicating if the likelihood ratio test is required. Valid only when two objects are entered
param	For the function logLik a list of parameters like defined in <a href="#">mmeln</a> , by default it is taken from the mmeln object. For the cov.tsf function it is vector containing the distinct value of the covariance as defined in the <a href="#">mmeln</a> function
type	Type of covariance as defined in <a href="#">mmeln</a>
p	Rank of covariance matrix
se.estim	Type of estimator. The default is MLR based on the information matrix define as $Ir^{(-1)}=I^{(-1)}IeI^{(-1)}$ . The other choices are the Observational information matrix "ML" and the Empirical information matrix based on the cross product of the gradient of the logLikelihood "ML.E"

**Details**

The function plot draws X\$G lines showing the expected value. The function logLik gives the log(Likelihood) of a model. The function anova compares mmeln models and gives the total number of parameters, the log(Likelihood), the AIC (Akaike information criterion), the BIC (Bayesian information criterion based on the number of observation) and the BIC2 (BIC based on the number of subjects). Optionally, the Likelihood ratio test is performed. The function print is used for solution given by the [estim.mmeln](#) function. The print method gives the number of iterations required for convergence and the statistics for the location, mixture and covariance parameters.

**Author(s)**

Charles-Édouard Giguère

**References**

McLachlan, G. & Peel, D. (2000), Finite mixture models, Wiley  
 Bernard D. Flury (1997), A first course in multivariate statistics, Springer  
 Pinheiro José C. & Bates Douglas M. (2000), Mixed-Effects Models in S and S-PLUS, Springer  
 M.S. Srivastava (2002), Methods of Multivariate Statistics, WILEY

**See Also**

[mmeln](#)

**Examples**

```
#### load an example.
data(exY)

### estimation of the parameters of the mixture
temps=1:3
mmeln1=mmeln(Y,G=2,form.loc=~factor(temps)-1,form.mel=~1,cov="CS")
mmeln2=mmeln(Y,G=2,form.loc=list(~temps,~I((temps-2)^2)),form.mel=~1,cov="CS")

mix1=estim(mmeln1,mu=list(rep(1,3),rep(2,3)),tau=c(0)
           ,sigma=list(c(1,.4),c(1,.4)),iterlim=100,tol=1e-6)

mix2=estim(mmeln2,mu=list(c(2,1),c(5,-1)),tau=c(0)
           ,sigma=list(c(1,.4),c(1,.4)),iterlim=100,tol=1e-6)

mix1
mix2

anova(mix1,mix2)
plot(mix1,main="Mixture of multivariate normal")
plot(mix2,main="Mixture of multivariate normal")
```

---

post.mmeln,entropy.mmeln

*Posterior probabilities, entropy for mmeln object*

---

**Description**

Compute the posterior probabilities of membership in each group of the mixture

**Usage**

```
## S3 method for class 'mmeln'
post(X,...,mu=X$param$mu,tau=X$param$tau,sigma=X$param$sigma)
## S3 method for class 'mmeln'
entropy(X,...)
```

**Arguments**

X	An object of type mmeln containing the design of the model.
...	These parameters are useless
mu	Location parameters. By default, those are taken from X
tau	Mixture parameters. By default, those are taken from X
sigma	Covariance parameters. By default, those are taken from X

**Details**

This procedure returns the posterior probabilities of membership in each groups or the entropy of the model. They were computed as described in McLachlan and Peel (2000). If the parameters X\$param is not null no further parameters are necessary, otherwise you have to give a value for mu, tau, sigma (this is mainly used inside the estim.mmeln function)

**Value**

Returns a matrix P with X\$N row and X\$G column where P[i,j] is the posterior probabilities of subject i being in the group j or the value of entropy.

**Author(s)**

Charles-Édouard Giguère

**References**

McLachlan, G. & Peel, D. (2000), Finite mixture models, Wiley

**See Also**

[estim.mmeln](#)

**Examples**

```
#### load an example.
data(exY)

### estimation of the parameters of the mixture
temps <- factor(1:3)
mmeln1 <- mmeln(Y, G = 2, form.loc = ~temps - 1, form.mel = ~1, cov = "CS")
mix1 <- estim(mmeln1, mu = list(rep(1,3),rep(2,3)), tau = c(0),
             sigma = list(c(1, .4), c(1, .4)), iterlim = 100, tol = 1e-6)
post(mix1)
entropy(mix1)
```

# Index

\*Topic **datasets**  
   exY, 6  
 \*Topic **density**  
   dmnorm, 3  
 \*Topic **mixture**  
   estim, 4  
   mmeln, 6  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln, 8  
   post.mmeln, entropy.mmeln, 10  
 \*Topic **mmeln**  
   estim, 4  
   mmeln, 6  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln, 8  
   post.mmeln, entropy.mmeln, 10  
 \*Topic **multivariate normal**  
   dmnorm, 3  
 \*Topic **multivariate**  
   estim, 4  
   mmeln, 6  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln, 8  
   post.mmeln, entropy.mmeln, 10  
 \*Topic **normal**  
   estim, 4  
   mmeln, 6  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln, 8  
   post.mmeln, entropy.mmeln, 10  
 \*Topic **package**  
   mmeln-package, 2  
 anova.mmeln  
   (plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln), 8  
 cov.tsf  
   (plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln), 8  
   post.mmeln, entropy.mmeln, 10  
   post.mmeln, entropy.mmeln, 10  
   covNA.wt (estim), 4  
   dmnorm, 3  
   dnorm, 3  
   eigen, 3  
   entropy (post.mmeln, entropy.mmeln), 10  
   estim, 4  
   estim.mmeln, 7–9, 11  
   estimloc.disp.CS1 (estim), 4  
   estimloc.disp.IND1 (estim), 4  
   estimmelnCS1 (estim), 4  
   estimmelnIND1 (estim), 4  
   exY, 6  
   I.CS1 (estim), 4  
   I.IND1 (estim), 4  
   IE.CS1 (estim), 4  
   IE.IND1 (estim), 4  
   logit (estim), 4  
   logLik.mmeln  
   mmeln, (plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln), 8  
   mmeln, 4, 6, 9, 10  
   mmeln-package, 2  
   mmeln.package, 5  
   mmelnr package (mmeln-package), 2  
   multnm  
   (plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln), 8  
   pfQ.intermediate.CS1 (estim), 4  
   plot.mmeln  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln), 8  
   plot.mmeln, logLik.mmeln, anova.mmeln, print.mmeln, 8  
   post.mmeln, entropy.mmeln, 10  
   post.mmeln, entropy.mmeln, 10

print.mmelnSOL  
    (plot.mmeln,logLik.mmeln,anova.mmeln,print.mmeln),  
    8

SEXE (exY), 6

Xinv (estim), 4

Y (exY), 6