# Package 'npmr' 

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npmr-package Nuclear penalized multinomial regression

## Description

As an alternative to an 11- or 12-penalty on multinomial logistic regression, this package fits multinomial regression with a penalty on the nuclear norm of the fitted regression coefficient matrix. The result is often a matrix of reduced rank, leveraging structure among the response classes so that the likelihood of one class informs the likelihood of other classes. Proximal gradient descent is used to solve the NPMR optimization problem.

## Details

| Package: | npmr |
| :--- | :--- |
| Type: | Package |
| Version: | 1.0 |
| Date: | $2016-02-05$ |
| License: | What license is it under? |

The primary functions in the package are npmr, which solves nuclear penalized multinomial regression for a sequence of input values for the regularization parameter lambda, and cv.npmr, which chooses the optimal value of the regularization parameter lambda via cross validation. Both npmr and $\mathrm{cv} . \mathrm{npmr}$ have predict and plot methods.

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani
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## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## Examples

\# Fit NPMR to simulated data
$\mathrm{K}=5$
n = 1000
m = 10000
$\mathrm{p}=10$
$r=2$
\# Simulated training data

```
set.seed(8369)
A = matrix(rnorm(p*r), p, r)
C = matrix(rnorm(K*r), K, r)
B = tcrossprod(A, C) # low-rank coefficient matrix
X = matrix(rnorm(n*p), n, p) # covariate matrix with iid Gaussian entries
eta = X
P = exp(eta)/rowSums(exp(eta))
Y = t(apply(P, 1, rmultinom, n = 1, size = 1))
# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, n = 1, size = 1))
# Fit NPMR for a sequence of lambda values without CV:
fit2 = npmr(X, Y, lambda = exp(seq(7, -2)))
# Print the NPMR fit:
fit2
# Produce a biplot:
plot(fit2, lambda = 20)
# Compute mean test error using the predict function (for each value of lambda):
getloss = function(pred, Y) {
    -mean(log(rowSums(Y*pred)))
}
apply(predict(fit2, Xtest), 3, getloss, Ytest)
```

Cross-validated nuclear penalized multinomial regression

## Description

Divide the training data into folds. Hold out each fold and fit NPMR for a range of regularization values on the remaining data, testing the result on the held-out fold. After the optimal value of the regularization parameter is determined, fit NPMR with this tuning parameter to the whole training set.

## Usage

```
cv.npmr(X, Y, lambda = exp(seq(7, -2)), s = 0.1/max (X), eps = 1e-06,
    group = NULL, accelerated = TRUE, B.init = NULL, b.init = NULL,
    foldid = NULL, nfolds = 10)
```


## Arguments

$x$ Covariate matrix. May be in sparse form from Matrix package

Y
eps Convergence threshold. When relative change in the objective function after an interation drops below this threshold, algorithm halts.
group Vector of length equal to number of variables (ncol(X) and nrow(B)). Variables in the same group indexed by a POSITIVE integer will be penalized together (the nuclear norm of the sub-matrix of the regression coefficients will be penalized). Variables without positive integers will NOT be penalized. Default is NULL, which means there are no sub-groups; nuclear norm of entire coefficient matrix is penalized.
accelerated Logical. Should accelerated proximal gradient descent be used? Default is TRUE.
B. init Initial value of the regression coefficient matrix for proximal gradient descent
b.init Initial value of the regression intercept vector for proximal gradient descent
foldid Vector of length equal to nrow(X). Specifies folds for cross validation.
nfolds Number of folds for cross validation. Ignored if foldid is specified. Default is 10.

## Value

An object of class "cv.npmr" with values:

| call | the call that produced this object |
| :--- | :--- |
| error | A vector of total cross validation error for each value of lambda |
| fit | An object of class npmr fitted to the entire training data using lambda.min |
| lambda.min | The value of lambda with minimum cross validation error <br> lambda |
| The input sequence of regularization parameter values for which cross validation <br> error was calculated |  |
| n | number of rows in the input covariate matrix $X$ |

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also <br> ```npmr, predict.cv.npmr, print.cv.npmr, plot.cv.npmr```

## Examples

\# Fit NPMR to simulated data

$$
K=5
$$

$$
n=1000
$$

$$
m=10000
$$

$$
p=10
$$

$$
r=2
$$

\# Simulated training data
set.seed(8369)
$\mathrm{A}=\operatorname{matrix}(r n o r m(p * r), p, r)$
$C=\operatorname{matrix}(\operatorname{rnorm}(K * r), K, r)$
$B=\operatorname{tcrossprod}(A, C) \quad \#$ low-rank coefficient matrix
X = matrix(rnorm(n*p), n, p) \# covariate matrix with iid Gaussian entries
eta $=\mathrm{X}$
$P=\exp (e t a) /$ rowSums $(\exp (e t a))$
$\mathrm{Y}=\mathrm{t}(\operatorname{apply}(\mathrm{P}, 1$, rmultinom, $\mathrm{n}=1$, size = 1))
fold $=\operatorname{sample}(\operatorname{rep}(1: 10$, length $=\operatorname{nrow}(X)))$
\# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, $\mathrm{n}=1$, size = 1))
\# Fit NPMR for a sequence of lambda values without CV:
fit2 $=$ cv.npmr(X, Y, lambda $=\exp (\operatorname{seq}(7,-2))$, foldid $=$ fold)
\# Print the NPMR fit:
fit2
\# Produce a biplot:
plot(fit2)
\# Compute mean test error using the predict function:
-mean(log(rowSums(Ytest*predict(fit2, Xtest))))

## Description

Computes the log-likelihood of the fitted regression parameters given the data observed. Intended for internal use only.

## Usage

$\log L(B, b, X, Y)$

## Arguments

| B | Regression coefficient matrix |
| :--- | :--- |
| b | Regression intercept vector |
| X | Covariate matrix |
| Y | Multinomial response matrix |

## Value

The log-likelihood of $B$ and $b$ given $X$ and $Y$

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

See Also
nuclear, objective

## npmr Nuclear penalized multinomial regression

## Description

Fit a multinomial logistic regression model for a sequence of regularization parameters penalizing the nuclear norm of the regression coefficient matrix.

## Usage

npmr (X, Y, lambda $=\exp (\operatorname{seq}(7,-2)), s=0.1 / \max (X), \mathrm{eps}=1 \mathrm{e}-06$, group $=$ NULL, accelerated $=$ TRUE, B.init $=$ NULL, b.init $=$ NULL, quiet $=$ TRUE)

## Arguments

| X | Covariate matrix. May be in sparse form from Matrix package <br> Multinomial reponse. May be (1) a vector of length equal to nrow(X), which <br> will be interpreted as a factor, with levels representing response classes; or (2) <br> a matrix with nrow(Y) = nrow(X), and each row has exactly one 1 representing <br> the response class for that observation, with the remaining entries of the row <br> being zero. <br> Vector of regularization parameter values for penalizing nuclear norm. Default <br> is a wide range of values. We suggest that the user choose this sequence via trial <br> and error. If the model takes too long to fit, try larger values of lambda. |
| :--- | :--- |
| lambda | Step size for proximal gradient descent |
| sonvergence threshold. When relative change in the objective function after an |  |
| eps | interation drops below this threshold, algorithm halts. |
| group | Vector of length equal to number of variables (ncol(X) and nrow(B)). Variables <br> in the same group indexed by a POSITIVE integer will be penalized together <br> (the nuclear norm of the sub-matrix of the regression coefficients will be pe- |
| nalized). Variables without positive integers will NOT be penalized. Default is |  |

## Details

In multinomial regression (in contrast with Gaussian regression or logistic regression) there is a matrix of regression coefficients, not just a vector. NPMR fits a logistic multinomial regression with a penalty on the nuclear norm of this regression coefficient matrix B. Specifically, the objective is $-\log \operatorname{lik}(\mathrm{B}, \mathrm{b} \mid X, Y)+\operatorname{lambda*}\|\mathrm{B}\| \_*$
where II.I_* denotes the nuclear norm. This implementation solves the problem using proximal gradient descent, which iteratively steps in the direction of the negative gradient of the loss function and soft-thresholds the singular values of the result.
This function makes available the option, through the groups argument, of dividing the regression coefficient matrix into sub-matrices (by row) and penalizing the sum of the nuclear norms of these submatrices. Rows (correspond to variables) can be given no penalty in this way.

## Value

An object of class "npmr" with values:
call the call that produced this object


#### Abstract

B A 3-dimensional array, with dimensions ( $n \operatorname{col}(X), n \operatorname{col}(Y)$, length(lambda)). For each lambda, this array stores the regression coefficient matrix which solves the NPMR optimization problem for that value of lambda. b A matrix with $n \operatorname{col}(Y)$ rows and length(lambda) columns. Each column stores the regression intercept vector which solves the NPMR optimization problem for that value of lambda. objective A vector of length equal to the length of lambda, giving the value value of the objective for the solution corresponding to each value of lambda. lambda The input sequence of values for the regularization parameter, for each of which NPMR has been solved.


## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

cv.npmr, predict.npmr, print.npmr, plot.npmr

## Examples

```
# Fit NPMR to simulated data
K = 5
n = 1000
m = 10000
p = 10
r = 2
# Simulated training data
set.seed(8369)
A = matrix(rnorm(p*r), p, r)
C = matrix(rnorm(K*r), K, r)
B = tcrossprod(A, C) # low-rank coefficient matrix
X = matrix(rnorm(n*p), n, p) # covariate matrix with iid Gaussian entries
eta = X
P = exp(eta)/rowSums(exp(eta))
Y = t(apply(P, 1, rmultinom, n = 1, size = 1))
# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, n = 1, size = 1))
# Fit NPMR for a sequence of lambda values without CV:
```

nuclear

```
fit2 = npmr(X, Y, lambda = exp(seq(7, -2)))
# Print the NPMR fit:
fit2
# Produce a biplot:
plot(fit2, lambda = 20)
# Compute mean test error using the predict function (for each value of lambda):
getloss = function(pred, Y) {
        -mean(log(rowSums(Y*pred)))
}
apply(predict(fit2, Xtest), 3, getloss, Ytest)
```

    nuclear \(\quad\) Nuclear norm of a matrix
    
## Description

Returns the nuclear norm of a matrix, which is the sum of its singular values, obtained through a singular value decomposition. Intended for internal use only.

## Usage

## nuclear (B)

## Arguments

B
a matrix

## Value

the nuclear norm of the matrix

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## See Also

logL, objective, objectiveFast

```
objective NPMR objective function
```


## Description

Return the objective function of the data and the fitted parameters for nuclear penalized multinomial regression. The objective is the sum of the negative log-likelihood and the product of the regularization parameter and nuclear norm of the fitted regression coefficient matrix. Intended for internal use only.

## Usage

objective(B, b, X, Y, lambda)

## Arguments

B
b fitted regression intercept vector
$\mathrm{X} \quad$ covariate matrix
Y multinomial response matrix
lambda regularization parameter (maybe be a vector of values)

## Value

a vector of objective values for the NPMR optimization problem, one for each value of lambda

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

logL, nuclear, objectiveFast

## Description

Computes the objective function for NPMR more quickly than objective by leveraging precomputed fitted values, which is the bottleneck in computing the objective. Intended for internal use only.

## Usage

objectiveFast(B, P, W, lambda)

## Arguments

B fitted regression coefficient matrix
P matrix of fitted multinomial class probabilities
$W \quad$ vector containing indices of $P$ which correspond to observed data
lambda regularization parameter (maybe be a vector of values)

## Value

a vector of objective values for the NPMR optimization problem, one for each value of lambda

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

objective, nuclear, PGDnpmr

PGDnpmr Proximal gradient descent for nuclear penalized multinomial regression

## Description

Iterates steps of proximal gradient descent until convergence, by repeatedly taking steps in the direction of the negative of the gradient and soft-thresholding the singular values of the result. Intended for internal use only.

## Usage

PGDnpmr (B, b, X, Y, lambda, s, group = NULL, accelerated = TRUE, eps = 1e-07, maxit $=1 \mathrm{e}+05$, quiet $=$ TRUE)

## Arguments

B Initial regression coefficient matrix
b Initial intercept vector
X Covariate matrix. May be in sparse form from Matrix package
Y Response matrix. Each row has exactly one 1 indicating response category for that observation. All other entries are zero.
lambda Vector of regularization parameter values for penalizing nuclear norm
s Step size for proximal gradient descent
group Vector of length equal to number of variables ( $n \operatorname{col}(X)$ and nrow(B)). Variables in the same group indexed by a POSITIVE integer will be penalized together (the nuclear norm of the sub-matrix of the regression coefficients will be penalized). Variables without positive integers will NOT be penalized. Default is NULL, which means there are no sub-groups; nuclear norm of entire coefficient matrix is penalized.
accelerated Logical. Should accelerated proximal gradient descent be used? Default is TRUE.
eps Convergence threshold. When relative change in the objective function after an interation drops below this threshold, algorithm halts.
maxit Maximum number of iterations for proximal gradient descent.
quiet Logical. Should output be silenced? If not, print the value of the objective function after each step of proximal gradient descent. Perhaps useful for debugging. Default is TRUE.

## Value

B Optimal value of the regression coeffient matrix at convergence
b Optimal value of the regression intercept vector at convergence
objectivePath Vector showing the value of the objective function at each step in proximal gradient descent
time Time taken until convergence

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

npmr, prox, objective, objectiveFast
plot.cv.npmr Visualize the regression coefficient matrix fit by cross-validated NPMR

## Description

Plots features (in orange) by their weights on the first two latent variables in the singular value decomposition of the regression coefficient matrix. Plots response classes (as blue arrows) by their loadings on the first two latent variables. Does this for the regression coefficient matrix fit with the value of lambda that led to the minimum cross validation error among all those tried.

## Usage

\#\# S3 method for class 'cv.npmr'
plot(x, feature.names = TRUE, ...)

## Arguments

$x \quad$ an object of class cv. npmr
feature. names logical. Should the names of the covariates be used in the plot? If FALSE, use standard plotting symbol (pch=1) instead.
.. additional arguments to be passed to plot

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

cv.npmr, plot.npmr

## Examples

\# Fit NPMR to simulated data
$K=5$
$n=1000$
$m=10000$
$p=10$
$r=2$
\# Simulated training data
set.seed(8369)
$A=\operatorname{matrix}(r n o r m(p * r), p, r)$
$C=\operatorname{matrix}(r \operatorname{norm}(K * r), K, r)$
$B=\operatorname{tcrossprod}(A, C) \quad$ \# low-rank coefficient matrix
$X=\operatorname{matrix}(r n o r m(n * p), n, p) \quad \#$ covariate matrix with iid Gaussian entries
eta $=X$
$P=\exp (e t a) / r o w S u m s(\exp (e t a))$
$Y=t(\operatorname{apply}(P, 1$, rmultinom, $n=1$, size $=1))$
fold $=\operatorname{sample}(\operatorname{rep}(1: 10$, length $=\operatorname{nrow}(X)))$
\# Simulate test data
Xtest $=\operatorname{matrix}($ rnorm $(m * p), m, p)$
etatest = Xtest
Ptest $=\exp ($ etatest $) /$ rowSums $(\exp ($ etatest $))$
Ytest $=\mathrm{t}(\operatorname{apply}$ (Ptest, 1 , rmultinom, $\mathrm{n}=1$, size $=1$ ))
\# Fit NPMR for a sequence of lambda values without CV:
fit2 $=c v . \operatorname{npmr}(X, Y, \operatorname{lambda}=\exp (\operatorname{seq}(7,-2))$, foldid $=$ fold)
\# Produce a biplot:
plot(fit2)
plot.npmr
Visualize the regression coefficient matrix fit by cross-validated NPMR

## Description

Plots features (in orange) by their weights on the first two latent variables in the singular value decomposition of the regression coefficient matrix. Plots response classes (as blue arrows) by their loadings on the first two latent variables. Does this for the regression coefficient matrix fit with the value of lambda closest among all those tried to the value of lambda specified.

## Usage

\#\# S3 method for class 'npmr'
plot ( $x$, lambda, feature.names = TRUE, ...)

## Arguments

X
lambda
feature.names
...
an object of class npmr
a single regularization parameter value
logical. Should the names of the covariates be used in the plot? If FALSE, use standard plotting symbol (pch=1) instead.
additional arguments to be passed to plot

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

npmr, plot.cv.npmr

## Examples

\# Fit NPMR to simulated data
$K=5$
$\mathrm{n}=1000$
$\mathrm{m}=10000$
$\mathrm{p}=10$
$r=2$
\# Simulated training data
set.seed(8369)
$A=\operatorname{matrix}(r \operatorname{norm}(p * r), p, r)$
$C=\operatorname{matrix}(r n o r m(K * r), K, r)$
$B=\operatorname{tcrossprod}(A, C) \quad \#$ low-rank coefficient matrix
$X=\operatorname{matrix}(\operatorname{rnorm}(n * p), n, p) \quad \#$ covariate matrix with iid Gaussian entries
eta $=X$
$P=\exp (e t a) /$ rowSums (exp(eta))
$\mathrm{Y}=\mathrm{t}(\operatorname{apply}(\mathrm{P}, 1$, rmultinom, $\mathrm{n}=1$, size = 1))
\# Simulate test data
Xtest $=$ matrix (rnorm(m*p), m, p)
etatest = Xtest
Ptest $=\exp ($ etatest $) /$ rowSums $(\exp ($ etatest $))$
Ytest = t(apply(Ptest, 1, rmultinom, $\mathrm{n}=1$, size = 1))
\# Fit NPMR for a sequence of lambda values without CV:
fit2 $=\operatorname{npmr}(X, Y, \operatorname{lambda}=\exp (\operatorname{seq}(7,-2)))$
\# Produce a biplot:
plot(fit2, lambda = 20)

```
predict.cv.npmr Make predictions from a "cv.npmr" object
```


## Description

Return predicted reponse class probabilities from a cross-validated NPMR model, using the value of the regularization parameter that led to the minimum cross validation error

## Usage

\#\# S3 method for class 'cv.npmr'
predict(object, newx, ...)

## Arguments

| object | an object of class cv.npmr |
| :--- | :--- |
| newx | covariate matrix on which for which to make response class probability predic- <br> tions. Must have same number of columns as $X$ used original to fit object. |
| $\ldots$ | ignored |

## Value

a matrix giving the predicted probability that each row of newx belongs to each class, corresponding the value of the regularization parameter that led to minimum cross validation error.

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

cv.npmr, predict.npmr

## Examples

\# Fit NPMR to simulated data

K = 5
n = 1000
$\mathrm{m}=10000$
$\mathrm{p}=10$
$r=2$

```
# Simulated training data
set.seed(8369)
A = matrix(rnorm(p*r), p, r)
C = matrix(rnorm(K*r), K,r)
B = tcrossprod(A, C) # low-rank coefficient matrix
X = matrix(rnorm(n*p), n, p) # covariate matrix with iid Gaussian entries
eta = X
P = exp(eta)/rowSums(exp(eta))
Y = t(apply(P, 1, rmultinom, n = 1, size = 1))
fold = sample(rep(1:10, length = nrow(X)))
# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, n = 1, size = 1))
# Fit NPMR for a sequence of lambda values without CV:
fit2 = cv.npmr(X, Y, lambda = exp(seq(7, -2)), foldid = fold)
# Compute mean test error using the predict function:
-mean(log(rowSums(Ytest*predict(fit2, Xtest))))
```

predict.npmr Make predictions from a "npmr" object

## Description

Return predicted reponse class probabilities from a fitted NPMR model, for each value of lambda on which the NPMR model was originally fit.

## Usage

\#\# S3 method for class 'npmr' predict(object, newx, ...)

## Arguments

object an object of class npmr
newx covariate matrix on which for which to make response class probability predictions. Must have same number of columns as $X$ used original to fit object.
... ignored

## Value

a 3-dimensional array, with dimensions (nrow(newx), ncol(Y), length(lambda)). For each lambda, this array stores for that value of lambda the predicted response class probabilites for each observation.

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

npmr, predict.cv.npmr

## Examples

```
# Fit NPMR to simulated data
K = 5
n = 1000
m = 10000
p = 10
r=2
# Simulated training data
set.seed(8369)
A = matrix(rnorm(p*r), p, r)
C = matrix(rnorm(K*r), K, r)
B = tcrossprod(A, C) # low-rank coefficient matrix
X = matrix(rnorm(n*p), n, p) # covariate matrix with iid Gaussian entries
eta = X
P = exp(eta)/rowSums(exp(eta))
Y = t(apply(P, 1, rmultinom, n = 1, size = 1))
# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, n = 1, size = 1))
# Fit NPMR for a sequence of lambda values without CV:
fit2 = npmr(X, Y, lambda = exp(seq(7, -2)))
# Compute mean test error using the predict function (for each value of lambda):
getloss = function(pred, Y) {
    -mean(log(rowSums(Y*pred)))
}
apply(predict(fit2, Xtest), 3, getloss, Ytest)
```

```
print.cv.npmr summarize a "cv.npmr" object
```


## Description

Print (1) the call that produced the $\mathrm{cv} . \mathrm{npmr}$ object; (2) the value of the regularization parameter lambda that led to the minimum cross validation error; (3) the rank of the fitted regression coefficient matrix; and (4) the per-observation cross validation error using the optimal lambda.

## Usage

\#\# S3 method for class 'cv.npmr'
print(x, ...)

## Arguments

$\begin{array}{ll}\mathrm{X} & \text { an object of class } \mathrm{CV} . \mathrm{npmr} \\ \ldots & \text { ignored }\end{array}$

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

cv.npmr, print.npmr

## Examples

\# Fit NPMR to simulated data
$\mathrm{K}=5$
$\mathrm{n}=1000$
$\mathrm{m}=10000$
$\mathrm{p}=10$
$r=2$
\# Simulated training data
set.seed(8369)
$A=\operatorname{matrix}(r n o r m(p * r), p, r)$
$C=\operatorname{matrix}(r n o r m(K * r), K, r)$
$B=\operatorname{tcrossprod}(A, C) \quad \#$ low-rank coefficient matrix
X = matrix (rnorm(n*p), n, p) \# covariate matrix with iid Gaussian entries
eta $=\mathrm{X}$

```
P = exp(eta)/rowSums(exp(eta))
Y = t(apply(P, 1, rmultinom, n = 1, size = 1))
fold = sample(rep(1:10, length = nrow (X)))
# Simulate test data
Xtest = matrix(rnorm(m*p), m, p)
etatest = Xtest
Ptest = exp(etatest)/rowSums(exp(etatest))
Ytest = t(apply(Ptest, 1, rmultinom, n = 1, size = 1))
# Fit NPMR for a sequence of lambda values without CV:
fit2 = cv.npmr(X, Y, lambda = exp(seq(7, -2)), foldid = fold)
# Print the NPMR fit:
fit2
```

print.npmr Summarize a "npmr" object

## Description

Print the call that produced the npmr object and a dataframe showing, for each value of the regularization parameter on which the NPMR object was fit, the rank of the resulting regression coefficient matrix and the corresponding value of the NPMR objective function.

## Usage

\#\# S3 method for class 'npmr'
print(x, ...)

## Arguments

$$
\begin{array}{ll}
x & \text { an object of class npmr } \\
\ldots & \text { ignored }
\end{array}
$$

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Scott Powers, Trevor Hastie and Rob Tibshirani (2016). "Nuclear penalized multinomial regression with an application to predicting at bat outcomes in baseball." In prep.

## See Also

npmr, print.cv.npmr

## Examples

\# Fit NPMR to simulated data
$\mathrm{K}=5$
$\mathrm{n}=1000$
$\mathrm{m}=10000$
$\mathrm{p}=10$
$r=2$
\# Simulated training data
set.seed(8369)
$A=\operatorname{matrix}(r \operatorname{norm}(p * r), p, r)$
$C=\operatorname{matrix}(r \operatorname{norm}(K * r), K, r)$
$B=\operatorname{tcrossprod}(A, C) \quad \#$ low-rank coefficient matrix
$X=\operatorname{matrix}(r n o r m(n * p), n, p) \quad \#$ covariate matrix with iid Gaussian entries
eta $=X$
$P=\exp (e t a) /$ rowSums (exp(eta))
Y = t(apply (P, 1, rmultinom, $\mathrm{n}=1$, size = 1))
\# Simulate test data
Xtest $=$ matrix $(r n o r m(m * p), m, p)$
etatest $=$ Xtest
Ptest $=\exp ($ etatest $) /$ rowSums (exp(etatest))
Ytest = t(apply (Ptest, 1, rmultinom, $\mathrm{n}=1$, size = 1))
\# Fit NPMR for a sequence of lambda values without CV:
fit2 $=\operatorname{npmr}(X, Y, \operatorname{lambda}=\exp (\operatorname{seq}(7,-2)))$
\# Print the NPMR fit:
fit2
prox
Proximal operator for nuclear norm

## Description

Return the value of the proximal operator of the nuclear norm (scaled by threshold) applied to a matrix

## Usage

prox (B, threshold, group)

## Arguments

B
threshold
matrix
scaling factor applied to the nuclear norm. In proximal gradient descent for NPMR, this is the product of the stepsize and the regularization parameter lambda
group Vector of length equal to number of variables, i.e. nrow(B). Variables in the same group indexed by a POSITIVE integer will be penalized together (the nuclear norm of the sub-matrix of the regression coefficients will be penalized). Variables without positive integers will NOT be penalized. Default is NULL, which means there are no sub-groups; nuclear norm of entire coefficient matrix is penalized.

## Value

the value of the proximal operator of the nuclear norm (scaled by threshold) applied to $B$

## Author(s)

Scott Powers, Trevor Hastie, Rob Tibshirani

## References

Neal Parikh and Stephen Boyd (2013) "Proximal algorithms." Foundations and Trends in Optimization 1, 3:123-231.

## See Also

nuclear, PGDnpmr

Vowel Vowel Recognition

## Description

Speaker independent recognition of the eleven steady state vowels of British English using a specified training set of lpc derived $\log$ area ratios.

## Format

A data frame with 990 observations on the following 12 variables.
y Class label indicating vowel spoken
subset a factor with levels test train
x .1 a numeric vector
x .2 a numeric vector
x. 3 a numeric vector
x .4 a numeric vector
x .5 a numeric vector
$x .6$ a numeric vector
$x .7$ a numeric vector
x .8 a numeric vector
x. 9 a numeric vector
x .10 a numeric vector

## Details

The speech signals were low pass filtered at 4.7 kHz and then digitised to 12 bits with a 10 kHz sampling rate. Twelfth order linear predictive analysis was carried out on six 512 sample Hamming windowed segments from the steady part of the vowel. The reflection coefficients were used to calculate $10 \log$ area parameters, giving a 10 dimensional input space. For a general introduction to speech processing and an explanation of this technique see Rabiner and Schafer [RabinerSchafer78].
Each speaker thus yielded six frames of speech from eleven vowels. This gave 528 frames from the eight speakers used to train the networks and 462 frames from the seven speakers used to test the networks.
The eleven vowels, along with words demonstrating their sound, are: i (heed) I (hid) E (head) A (had) a: (hard) Y (hud) O (hod) C: (hoard) U (hood) u: (who'd) 3: (heard)

## Source

https://archive.ics.uci.edu/ml/machine-learning-databases/undocumented/connectionist-bench/vowel/

## References

D. H. Deterding, 1989, University of Cambridge, "Speaker Normalisation for Automatic Speech Recognition", submitted for PhD.

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