# Package 'sglasso'

December 5, 2019

Type Package

Title Lasso Method for RCON(V,E) Models
Version 1.2.4
<b>Date</b> 2019-12-05
Author Luigi Augugliaro
Maintainer Luigi Augugliaro <luigi.augugliaro@unipa.it></luigi.augugliaro@unipa.it>
<b>Depends</b> Matrix, igraph, R (>= 3.6)
Imports methods, graphics, grDevices
Encoding latin1
<b>Description</b> RCON(V, E) models are a kind of restriction of the Gaussian Graphical Models defined by a set of equality constraints on the entries of the concentration matrix. 'sglasso' package implements the structured graphical lasso (sglasso) estimator proposed in Abbruzzo et al. (2014) for the weighted 11-penalized RCON(V, E) model. Two cyclic coordinate algorithms are implemented to compute the sglasso estimator, i.e. a cyclic coordinate minimization (CCM) and a cyclic coordinate descent (CCD) algorithm.
<b>License</b> GPL (>= 2)
LazyLoad yes
<pre>URL http://www.unipa.it/persone/docenti/a/luigi.augugliaro</pre>
NeedsCompilation yes
Repository CRAN
<b>Date/Publication</b> 2019-12-05 09:10:02 UTC
R topics documented:
sglasso-package       2         fglasso       3         gplot       5         gplot.fglasso       6         gplot.sglasso       7         Kh       8

2 sglasso-package

sglas	sso-package	Lasso Mo	ethod for RCC	ON(V, E) Models	
Index					20
	1 0				
	C				

### **Description**

RCON(V, E) models (Hojsgaard, et al., 2008) are a kind of restriction of the Gaussian Graphical Models defined by a set of equality constraints on the entries of the concentration matrix. sglasso package implements the structured graphical lasso (sglasso) estimator proposed in Abbruzzo et al. (2014) for the weighted 11-penalized RCON(V, E) model. Two cyclic coordinate algorithms are implemented to compute the sglasso estimator, i.e. a cyclic coordinate minimization (CCM) and a cyclic coordinate descent (CCD) algorithm.

#### **Details**

Package: sglasso
Type: Package
Version: 1.2.4
Date: 2019-12-05
License: GPL (>=2)

### Author(s)

Luigi Augugliaro

Maintainer:

Luigi Augugliaro < luigi.augugliaro@unipa.it>

### References

Abbruzzo, A., Augugliaro, L., Mineo, A. M. and Wit, E. C. (2014) Cyclic coordinate for penalized Gaussian Graphical Models with symmetry restrictions. In *Proceeding of COMPSTAT 2014 - 21th International Conference on Computational Statistics*, Geneva, August 19-24, 2014.

Hojsgaard, S. and Lauritzen, S. L. (2008) Graphical gaussian models with edge and vertex symmetries. *J. Roy. Statist. Soc. Ser. B.*, Vol. **70(5)**, 1005–1027.

fglasso 3

fglasso

L1-penalized Factorial Graphical Lasso Model

### **Description**

Fit the weight 11-penlized factorial dynamic Gaussian Graphical Model.

### Usage

```
fglasso(S, model, tp, p, ...)
```

### **Arguments**

S the empirical variance/covariance matrix;

model a list or a matrix used to specify the factorial dynamic Gaussian Graphical Model

(see Details);

tp number of time points;

p number of random variables observed for each time point;

... further arguments passed to sglasso.

#### **Details**

The factorial dynamic Gaussian Graphical Model (Abbruzzo et al., 2015) is a special kind of RCON(V, E) model (Hojsgaard, et al.,2008) proposed to study dynamic networks. Let  $X_t = (X_{it}, \ldots, X_{it})'$  be a p-dimensional random variable at time t. Assuming that  $X = (X_1', \ldots, X_T')$  follows a multivariate normal distribution, the concentration matrix K has the following block structure

$$K = \begin{pmatrix} K_{1,1} & K_{1,2} & \dots & K_{1,T} \\ K_{2,1} & K_{2,2} & \dots & K_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ K_{T,1} & K_{T,2} & \dots & K_{T,T} \end{pmatrix},$$

where  $K_{t,t}$  give information about the conditinal independence structure among the p random variables at time t, and  $K_{t,t+h}$  give information about the conditional independence structure between  $X_t$  and  $X_{t+h}$ . An interpretation of the elements of the submatrices  $K_{t,t+h}$  brings to the notion of natural structure, i.e.,

$$K_{t,t+h} = \begin{pmatrix} k_{1,1}^{t,t+h} & 0 & \dots & 0 \\ 0 & k_{2,2}^{t,t+h} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & k_{p,p}^{t,t+h} \end{pmatrix} + \begin{pmatrix} 0 & k_{1,2}^{t,t+h} & \dots & k_{1,p}^{t,t+h} \\ k_{2,1}^{t,t+h} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k_{p,1}^{t,t+h} & k_{p,2}^{t,t+h} & \dots & 0 \end{pmatrix}.$$

The entries of the first matrix are called **self-self conditinal dependences** at temporal lag h and represents the (negative) self-similarity of a given random variable across different time points. The entries of the second matrix are the **conditional dependence** among the p random variables. To make the interpretation of the results more relevant and, at the same time, reduce the number of parameters, the authors propose the following equality constraints:

4 fglasso

	$k_{i,i}^{t,t+h}$	effect	R code		$k_{i,j}^{t,t+h}$	effect	R code
i.	0	zero	"."	iv.	0	zero	"."
ii.	$s^h$	costant	"c"	ii.	$n^h$	costant	"c"
iii.	$s_i^h$	unit	"u"	iii.	$n_i^h$	unit	"u"
iv.	$s^{t,h}$	time	"t"	iv.	$n^{t,h}$	time	"t"
v.	$s_i^{t,h}$	interaction	"ut"	v.	$n_{i,j}^{t,h}$	interaction	"ut"

Argument model is used to specify the restrinctions previously describted. This argument can be a named list or a matrix with dimension  $nlag \times 2$ , where  $nlag \leq$  tp. To gain more insight, suppose that we want to model only the sub-matrices  $K_{t,t}$  and  $K_{t,t+1}$ , i.e., the sub-matrices corresponding to the temporal lag zero and one. A possible R code is

```
model.mat <-matrix("",nrow = 2,ncol = 2)
rownames(model.mat) <-c("lag0","lag1")
colnames(model.mat) <-c("s","n")
model.mat[1,] <-c("c","ut")
model.mat[2,] <-c("t",".")</pre>
```

In this example we are modelling the diagonal elements of the sub-matrices  $K_{t,t}$  with the constant effect while the off-diagonal elements are modelled by the interaction effect. In the same way, the diagonal elements of the sub-matrices  $K_{t,t+1}$  are modelled by the time effect while the remaining elements are equal to zero. The fglasso function passes the matrix model.mat to the internal function fglasso\_model2mask, i.e.,

```
mask <-fglasso_model2mask(model.mat,tp = 3,p = 3)</pre>
```

which returns the mask used in sglasso to fit the specified factorial dynamic Gaussian Graphical model. The same model can be specified by the following named list

```
model.list < -list(lag0 = c(s = "c", n = "ut"), lag1 = c(s = "t", n = "."))
```

See the example below for more details.

### Value

fglasso returns an obejet with S3 class "sglasso". See the corresponding manual for more details.

### Author(s)

Luigi Augugliaro

Maintainer: Luigi Augugliaro < luigi.augugliaro@unipa.it>

#### References

Wit, E. C. and Abbruzzo, A.(2015) Dynamic factorial graphical models for dynamic networks. *Network Science*, Vol. **3(1)**, 37–57

gplot 5

Abbruzzo, A., Augugliaro, L., Mineo, A.M. and Wit, E.C. (2014) Cyclic coordinate for penalized Gaussian Graphical Models with symmetry restrictions. In *Proceeding of COMPSTAT 2014 - 21th International Conference on Computational Statistics*. Geneva, August 19-24, 2014.

Hojsgaard, S. and Lauritzen, S.L. (2008) Graphical gaussian models with edge and vertex symmetries. *J. Roy. Statist. Soc. Ser. B.*, Vol. **70(5)**, 1005–1027.

### See Also

sglasso function.

### **Examples**

gplot

Plotting Sparse Graph

### **Description**

gplot is a generic function for plotting sparse graphs.

### Usage

```
gplot(object, ...)
```

### Arguments

```
object fitted sglasso/fglasso object;
... other parameters passed to gplot.sglasso or gplot.fglasso.
```

### **Details**

gplot is a generic function used to plot a graph estimated by sglasso or fglasso. See the method function gplot.sglasso or gplot.fglasso for more details about the specific arguments.

6 gplot.fglasso

### Author(s)

```
Luigi Augugliaro
```

Maintainer: Luigi Augugliaro < luigi.augugliaro@unipa.it>

### See Also

```
gplot.sglasso and gplot.fglasso method functions.
```

gplot.fglasso Plotting

Plotting Sparse Factorial Dynamic Gaussian Graphical Model

### **Description**

gplot.fglasso shows the sequence of graphs estimated by fglasso.

### Usage

```
## S3 method for class 'fglasso'
gplot(object, rhoid, tp = c(1, 2), sub.tp1, sub.tp2, cex.sub = 1,
    k = 1.5, layout = layout.circle, ...)
```

### **Arguments**

object	fitted fglasso object;
rhoid	an integer used to specificy the $\rho$ -value used to fit the fglasso model;
tp	a vector of length equal to two used to specify the time points of the two graphs that will be compared. By default the first two time points are used;
sub.tp1	sub title for the graph estimated at time point tp[1];
sub.tp2	sub title for the graph estimated at time point tp[2];
cex.sub	a numerical value giving the amount by which plotting sub titles should be magnified relateve to the default;
k	value used to specify the distance between the two graphs;
layout	a function or a matrix used to specify the layout of the graphs that will be plotted. By default the layout.circle function is used;
• • •	further graphical parameters used to plot the graphs. See package <b>igraph</b> for more details.

### **Details**

For a given value of the tuning parameter, specified by the argument rhoid, gplot.fglasso shows the graphs estimated at the time points tp[1] and tp[2]. By convention, the graph associated to the sub matrix  $K_{tp[1],tp[2]}$  is represented by a directed graph where a directed edge is drawn by an arrow from a vertex in the first graph pointing forwards a vertex in the second graph.

gplot.sglasso 7

### Value

gplot.fglasso returns a list with components:

graph.tp1	an object with class igraph representing the undirected graph estimated at the time point $tp[1]$ ;
graph.tp2	an object with class igraph representing the undirected graph estimated at the time point $tp[2]$ ;
graph.net	an object with class igraph representing the directed graph associated to the submatrix $K_{tp[1],tp[2]}$ ;
layout	the matrix used to specify the placement of the vertices.

### Author(s)

```
Luigi Augugliaro Maintainer: Luigi Augugliaro <luigi.augugliaro@unipa.it>
```

### See Also

fglasso function.

### **Examples**

gplot.sglasso

Plotting Sparse Graphs

### **Description**

```
gplot.sglasso shows the sequence of graphs estimated by sglasso.
```

### Usage

```
## S3 method for class 'sglasso'
gplot(object, rhoid, layout = layout.circle, ...)
```

8 Kh

### Arguments

object	fitted sglasso object;
rhoid	vector of integers used to specificy the $\rho$ -values used to fit the sglasso model. By default gplot.sglasso shows the sequence of graphs estimated by sglasso. Only topologically different graphs are plotted;
layout	a function or a matrix used to specify the layout of the graphs that will be plotted. By default the layout.circle function is used;
• • •	further graphical parameters used to plot the graphs. See package <b>igraph</b> for more details.

### **Details**

gplot.sglasso shows the sequence of topologically different graphs estimated by sglasso. To specify the layout of the graphs, the user can use any layout function available in the R package **igraph**. The user can also specify the placement of the vertices by a matrix with two columns and the same number of rows as the number of vertices.

### Author(s)

```
Luigi Augugliaro Maintainer: Luigi Augugliaro <luigi.augugliaro@unipa.it>
```

### See Also

sglasso function.

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X)/N
mask <- outer(1:p, 1:p, function(i,j) 0.5^abs(i-j))
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
gplot(out.sglasso_path)
gplot(out.sglasso_path, rhoid = 1:5)</pre>
```

Kh

Extract Sparse Structured Precision Matrices

### **Description**

Function Kh computes the sequence of sparse structured precision matrices estimated by sglasso function.

klev 9

### Usage

```
Kh(object, rho)
```

### **Arguments**

object fitted sglasso object;

rho a subset of the values of the tuning parameter used in sglasso to compute the

solution path. By default, the entire sequence of estimated sparse structured

precision matrices is returned.

#### Value

Kh returns a named list containing the sequence of estimated sparse structured precision matrices.

#### Author(s)

```
Luigi Augugliaro
Maintainer: Luigi Augugliaro < luigi .augugliaro@unipa.it>
```

### See Also

sglasso function.

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X) / N
mask <- outer(1:p, 1:p, function(i, j) 0.5^abs(i - j))
out.sglasso_path <- sglasso(S, mask, nrho = 5, tol = 1.0e-13)
out.sglasso_path
Kh(out.sglasso_path)
rho <- out.sglasso_path$rho[3]
out.sglasso_single <- sglasso(S, mask, nrho = 1, min_rho = rho,
    tol = 1.0e-13, algorithm = "ccm")
Kh(out.sglasso_single)</pre>
```

klcv

Cross-Validated Kullback-Leibler Divergence

### **Description**

Model selection criterion based on the leave-one-out cross-validated Kullback-Leibler divergence.

### Usage

```
klcv(object, X, scale = 1)
```

10 klcv

#### **Arguments**

object fitted sglasso/fglasso object;

X the matrix used to compute the empirical variance/covariance matrix. Its dimension is N × p, where p is the number of random variables and N is the samlpe

ize:

scale scalar value used to scale the estimated degrees-of-freedom. See below for more

details.

#### **Details**

klcv function implements the leave-one-out cross-validate Kullback-Leibler divergence criterion proposed in Vujacic et al. (2015). For  $l_1$ -penalized Gaussian Graphical Models this measure of goodness-of-fit has the following form

$$klcv(
ho) = -rac{\ell(\hat{K}(
ho))}{N} + rac{ extsf{scale}}{2N} gdf(\hat{K}(
ho)),$$

where  $\hat{K}(\rho)$  is the glasso estimate of the concentration matrix,  $\ell(\hat{K}(\rho))$  is the corresponding value of the log-likelihood function, scale is a scale factor for the complexity part, i.e.  $gdf(\hat{K}(\rho))$ , which is defined as

$$gdf(\hat{K}(\rho)) = \frac{1}{N-1} \sum_{k=1}^{N} vec\{(\hat{K}(\rho)^{-1} - S_k) \circ 1_{\rho}\}' vec[\hat{K}(\rho)\{(S - S_k) \circ 1_{\rho}\}\hat{K}(\rho)].$$

In the previous expression S is the empirical variance/covariance matrix,  $S_k = X_k X_k'$ ,  $1_\rho$  is a matrix with entries  $I(\hat{k}_{ij}(\rho) \neq 0)$  and  $\circ$  is the Hadamard product operator.

#### Value

rhoid

klcv returns an S3 object with calls klcv, i.e. a named list with the following components:

klcv the vector with the leave-one-out cross-validated Kullback-Leibler divergence; rho the rho-values used to compute the leave-one-out cross-validated Kullback-Leibler divergence; loglik a vector with the log-likelihood computed for the sequence of weighted 11penalized RCON(V, E); gdf a vector returning the generalized degrees-of-freedom; scale the scale value used to define the leave-one-out cross-validated Kullback-Leibler divergence; minimum value of the leave-one-out cross-validated Kullback-Leibler divermin.klcv rho.opt the rho-value corresponding to minimum leave-one-out cross-validated Kullback-Leibler divergence;

> the index of the rho-value identified by the leave-one-out cross-validated Kullback-Leibler divergence.

loglik 11

### Author(s)

Luigi Augugliaro

Maintainer: Luigi Augugliaro < luigi.augugliaro@unipa.it>

#### References

Vujacic, I., Abbruzzo, A. and Wit, E. C. (2015) A computationally fast alternative to cross-validation in penalized Gaussian graphical models. *J. Stat. Comput. Simul.* 

### See Also

```
sglasso, loglik functions and plot.klcv method.
```

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X) / N
mask <- outer(1:p, 1:p, function(i,j) 0.5^abs(i-j))
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
out.klcv <- klcv(out.sglasso_path, X)
out.klcv</pre>
```

loglik

Extract Log-Likelihood

### Description

This function extracts the log-likelihood for the sequence of weighted 11-penalized RCON(V, E) models estimated by sglasso function.

### Usage

```
loglik(object, N = 2)
```

### **Arguments**

object a fitted sglasso object;

sample size. Default value is 2 to remove the constant term in the log-likelihood

function. See below for more details.

12 loglik

### **Details**

Denoted with  $\psi = (\eta', \theta')'$  the parameter vector of the structured concentration matrix  $K(\psi)$ , the log-likelihood function of the RCON(V, E) model is equal, up to a constant, to the following expression

$$\ell(\psi) = \frac{N}{2} [\log \det K(\psi) - tr\{SK(\psi)\}],$$

where  $S=N^{-1}\sum_{i=1}^N X_iX_i^T$ , N is the sample size and  $X_i$  is the ith observed p-dimensional vector. Denoted with  $\hat{\psi}=(\hat{\eta}',\hat{\theta}')'$  the sglasso estimates, straightforward algebra shows that

$$\ell(\hat{\psi}) = \frac{N}{2} [\log \det K(\hat{\psi}) - p + \rho \sum_{m=1}^{S} w_m |\hat{\theta}_m|],$$

where  $\rho$  is the tuning parameter and  $w_m$  are the weights used to define the weighted 11-norm.

### Value

loglik returns a vector containing the log-likelihood computed for the sequence of weighted 11-penalized RCON(V, E).

### Author(s)

Luigi Augugliaro

Maintainer: Luigi Augugliaro < luigi .augugliaro@unipa.it>

#### See Also

summary.sglasso method and sglasso function.

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X) / N
mask <- outer(1:p, 1:p, function(i, j) 0.5^abs(i-j))
out.sglasso_path <- sglasso(S, mask, nrho = 5, tol = 1.0e-13)
out.sglasso_path
loglik(out.sglasso_path, N = N)
rho <- out.sglasso_path$rho[3]
out.sglasso_single <- sglasso(S, mask, nrho = 1, min_rho = rho,
    tol = 1.0e-13, algorithm = "ccm")
loglik(out.sglasso_single, N = N)</pre>
```

neisseria 13

### **Description**

This data set contains the gene expression data from a high-resolution time-course experiment besed on the sequenced Neisseria meningitidis serogroup strain B strain MC58. Specifically, the expression level of ten genes is measured at ten different time points. Each column is standardized to have zero mean and standard deviation equal to one.

### Usage

### Description

plot.klcv produces a plot to study the sequence of leave-one-out cross-validated Kullback-Leibler divergences computed by klcv.

### Usage

```
## S3 method for class 'klcv' plot(x, ...)
```

### **Arguments**

x fitted klcv object;

. . . other parameters to be passed through the plotting function.

### **Details**

This method function produces a plot showing the sequence of leave-one-out cross-validated Kullback-Leibler as function of the tuning parameter rho. The optimal value of the tuning parameter is identified by a vertical dashed line.

#### Author(s)

```
Luigi Augugliaro Maintainer: Luigi Augugliaro <luigi.augugliaro@unipa.it>
```

### See Also

klcv function.

14 plot.sglasso

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X) / N
mask <- outer(1:p, 1:p, function(i,j) 0.5^abs(i-j))
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
out.klcv <- klcv(out.sglasso_path, X)
plot(out.klcv)</pre>
```

plot.sglasso

Plot Method for the Weighted 11-Penalized RCON(V, E) Model

### **Description**

plot.sglasso produces two plots to study the sequence of models estimates by sglasso or fglasso.

### Usage

```
## S3 method for class 'sglasso'
plot(x, ...)
```

#### **Arguments**

x fitted sglasso/fglasso object;

... other parameters to be passed through the plotting function.

### **Details**

This function produces two different plots. The first one shows the path of the estimated parameters as function of the tuning parameter  $\rho$ . In the same way, the second plot shows the path of the weighted scores as function of  $\rho$ .

### Author(s)

```
Luigi Augugliaro Maintainer: Luigi Augugliaro <luigi.augugliaro@unipa.it>
```

#### See Also

sglasso function and summary.sglasso method.

sglasso 15

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X)/N
mask <- outer(1:p, 1:p, function(i,j) 0.5^abs(i-j))
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
plot(out.sglasso_path)</pre>
```

sglasso

Lasso Method for the RCON(V, E) Models

### **Description**

Fit the weighted 11-penalized *RCON(V, E)* models using a cyclic coordinate algorithm.

### Usage

### **Arguments**

S	the empirical variance/covariance matrix;
mask	a symmetric matrix used to specify the equality constraints on the entries of the concentration matrix. See the example bellow for more details;
W	a vector specifying the weights used to compute the weighted 11-norm of the parameters of the $RCON(V, E)$ model;
flg	a logical vector used to specify if a parameter is penalized, i.e., if $flg[i] = TRUE$ then the i-th parameter is penalized, otherwise ( $flg[i] = FALSE$ ) the maximum likelihood estimate is computed;
min_rho	last value of the sequence of tuning parameters used to compute the sglasso solution path. If nrho = 1, then min_rho is the value used to compute the sglasso estimate. Default value is 1.0e-02;
nrho	number of tuning parameters used to compute the sglasso solution path. Default is 50;
nstep	nonnegative integer used to specify the maximun number of iterations of the two cyclic coordinate algorithms. Default is 1.0e+05;
algorithm	character by means of to specify the algorithm used to fit the model, i.e., a cyclic coordinate descente (ccd) algorithm or a cyclic coordinate minimization (ccm) algorithm. Default is ccd;
truncate	at convergence all estimates below this value will be set to zero. Default is 1e-05;
tol	value used for convergence. Default value is 1.0e-05.

16 sglasso

### **Details**

The RCON(V, E) model (Hojsgaard et al., 2008) is a kind of restriction of the Gaussian Graphical Model defined using a coloured graph to specify a set of equality constraints on the entries of the concentration matrix. Roughly speaking, a coloured graph implies a partition of the vertex set into R disjoint subsets, called vertex colour classes, and a partition of the edge set into S disjoint subsets, called edge colour classes. At each vertex/edge colour class is associated a specific colour. If we denote by  $K = (k_{ij})$  the concentration matrix, i.e. the inverse of the variance/covariance matrix  $\Sigma$ , the coloured graph implies the following equality constraints:

- 1.  $k_{ii} = \eta_n$  for any index i belonging to the nth vertex colour class;
- 2.  $k_{ij} = \theta_m$  for any pair (i,j) belonging to the mth edge colour class.

Denoted with  $\psi = (\eta', \theta')'$  the (R+S)-dimensional parameter vector, the concentration matrix can be defined as

$$K(\psi) = \sum_{n=1}^{R} \eta_n D_n + \sum_{m=1}^{S} \theta_m T_m,$$

where  $D_n$  is a diagonal matrix with entries  $D_{ii}^n = 1$  if the index i belongs to the nth vertex colour class and zero otherwise. In the same way,  $T_m$  is a symmetrix matrix with entries  $T_{ii}^m = 1$  if the pair (i,j) belongs to the mth edge colour class. Using the previous specification of the concentration matrix, the structured graphical lasso (sglasso) estimator (Abbruzzo et al., 2014) is defined as

$$\hat{\psi} = \arg\max_{\psi} \log \det K(\psi) - tr\{Sk(\psi)\} - \rho \sum_{m=1}^{S} w_m |\theta_m|,$$

where S is the empirical variance/covariance matrix,  $\rho$  is the tuning parameter used to control the ammount of shrinkage and  $w_m$  are weights used to define the weighted  $\ell_1$ -norm. By default, the sglasso function sets the weights equal to the cardinality of the edge colour classes.

### Value

W

sglasso returns an obejet with S3 class "sglasso", i.e. a named list containing the following components:

call the call that produced this object; nν number of vertex colour classes; number of edge colour classes; ne the matrix of the sglasso estimates. The first nv rows correspond to the untheta penalized parameters while the remaining rows correspond to the weighted 11penalized parameters; the vector of weights used to define the weighted 11-norm;

df nrho-dimensional vector of the number of estimated nonzero parameters;

rho nrho-dimensional vector of the sequence of tuning parameters;

grd the matrix of the scores;

nonnegative integer used to specify the maximum number of iterations of the nstep

algorithms;

sglasso 17

number of tuning parameters used to compute the sglasso solution path; nrho the algorithm used to fit the model; algorithm the value used to set to zero the estimated parameters; truncate tol a nonnegative value used to define the convergence of the algorithms; S the empirical variace/covariance matrix used to compute the sglasso solution path; mask the mask used to define the equality constraints on the entries of the concentration matrix; number of interations of the algorithm; conv an integer value used to encode the warnings related to the algorithms. If conv = 0 the convergence has been achieved otherwise the maximum number of iter-

#### Author(s)

Luigi Augugliaro

Maintainer: Luigi Augugliaro < luigi.augugliaro@unipa.it>

ations has been achieved.

#### References

Abbruzzo, A., Augugliaro, L., Mineo, A. M. and Wit, E. C. (2014) Cyclic coordinate for penalized Gaussian Graphical Models with symmetry restrictions. In *Proceeding of COMPSTAT 2014 - 21th International Conference on Computational Statistics*, Geneva, August 19-24, 2014.

Hojsgaard, S. and Lauritzen, S. L. (2008) Graphical gaussian models with edge and vertex symmetries. *J. Roy. Statist. Soc. Ser. B.*, Vol. **70(5)**, 1005–1027.

#### See Also

```
summary.sglasso, plot.sglasso gplot.sglasso and methods.
```

The function Kh extracts the estimated sparse structured concentration matrices.

### **Examples**

18 summary.sglasso

```
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
mask

out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
out.sglasso_path

rho <- out.sglasso_path$rho[20]
out.sglasso <- sglasso(S, mask, nrho = 1, min_rho = rho, tol = 1.0e-13, algorithm = "ccm")
out.sglasso
out.sglasso_path$theta[, 20]
out.sglasso$theta[, 1]</pre>
```

summary.sglasso

Summarizing sglasso Fits

### Description

summary method for class "sglasso".

### Usage

#### **Arguments**

object fitted sglasso object;

N sample size;

k character/numeric argument used to specify the 'weight' of the complexity part

in the measure of goodness-of-fit used to select the best model (see below for

more details). Default is k = "bic";

digits significant digits in printout; ... additional print arguments.

#### **Details**

summary.sglasso gives us information about the sequence of models estimated by the sglasso estimator. To select the best model, summary method uses a measure of Goodness-of-Fit (GoF) defined as follows:

$$-2\ell(\hat{\psi}) + k \times df,$$

where  $\ell(\hat{\psi})$  is the log-likelihood of the estimated weighted 11-penalized RCON(V, E) model, df is the number of nonzero estimated parameters and k is a non-negative value used to weight the complexity part in the measure of goodness-of-fit. By default the summary method computes the BIC criterion to select the best model (k = "bic"). The AIC criterion can be easily computed

summary.sglasso 19

setting k = "aic". The user can also define other measures of goodness-of-fit specifying k as any non-negative value.

The output of the summary method is divided in two sections. First section shows the call producing the argument object followed by a data. frame. The column named rho shows the sequence of the  $\rho$  values used to compute the solution curve, while the column log-lik shows the corresponding values of the log-likelihood function. The remaining columns show the number of estimated non-zero parameters, the values of the GoF and the asscoated ranking of the estimated models. Finally, the second section shows the estimated parameters of the best model identified by the used GoF criterion. Informations about the algorithm and the corresponding convergence are also provided.

### Value

A list with components table and theta\_gof is silently returned. The table component is the data.frame previously described while the component theta\_gof is the vector of the estimated parameters corresponding to the best models identified by the GoF criterion.

#### Author(s)

```
Luigi Augugliaro
Maintainer: Luigi Augugliaro < luigi .augugliaro@unipa.it>
```

### See Also

sglasso and loglik functions.

### **Examples**

```
N <- 100
p <- 5
X <- matrix(rnorm(N * p), N, p)
S <- crossprod(X) / N
mask <- outer(1:p, 1:p, function(i,j) 0.5^abs(i-j))
mask[1,5] <- mask[1,4] <- mask[2,5] <- NA
mask[5,1] <- mask[4,1] <- mask[5,2] <- NA
out.sglasso_path <- sglasso(S, mask, tol = 1.0e-13)
summary(out.sglasso_path$rho[20]
out.sglasso <- sglasso(S, mask, nrho = 1, min_rho = rho, tol = 1.0e-13)
summary(out.sglasso, N)</pre>
```

## **Index**

```
*Topic datasets
    neisseria, 13
*Topic graph
    fglasso, 3
    sglasso, 15
    sglasso-package, 2
*Topic models
    fglasso, 3
    gplot, 5
    gplot.fglasso, 6
    gplot.sglasso, 7
    Kh, 8
    klcv, 9
    loglik, 11
    plot.klcv, 13
    plot.sglasso, 14
    sglasso, 15
    summary.sglasso, 18
*Topic multivariate
    fglasso, 3
    sglasso, 15
    sglasso-package, 2
*Topic package
    sglasso-package, 2
fglasso, 3, 7
gplot, 5
gplot.fglasso, 6, 6
gplot.sglasso, 6, 7, 17
Kh, 8, 17
klcv, 9, 13
loglik, 11, 11, 19
neisseria, 13
plot.klcv, 11, 13
plot.sglasso, 14, 17
print.klcv(klcv), 9
```

```
print.sglasso (sglasso), 15
sglasso, 5, 8, 9, 11, 12, 14, 15, 19
sglasso-package, 2
summary.sglasso, 12, 14, 17, 18
```