Package 'unbalhaar'

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Type Package

Title Function Estimation via Unbalanced Haar Wavelets

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Description Top-down and bottom-up algorithms
for nonparametric function estimation in Gaussian noise using
Unbalanced Haar wavelets.

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LazyLoad yes

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unbalhaar-package *Function estimation via Unbalanced Haar wavelets*

Description

The package implements top-down and bottom-up algorithms for nonparametric function estimation in Gaussian noise using Unbalanced Haar wavelets.

Details

Package:	unbalhaar
Type:	Package
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LazyLoad:	yes

The main functions of the package are `uh` and `uh.bu`.

Author(s)

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References

P. Fryzlewicz (2007) "Unbalanced Haar technique for nonparametric function estimation". *Journal of the American Statistical Association*, 102, 1318-1327.

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est.topdown <- uh(x)
est.bottomup <- uh.bu(x)
```

best.unbal.haar *Best top-down Unbalanced Haar decomposition*

Description

The function finds the "best" top-down Unbalanced Haar (UH) decomposition of the input vector `x`, according to a selection rule (`criterion`) which specifies which UH vector gets chosen at each scale and location.

Usage

```
best.unbal.haar(x, criterion = inner.prod.max)
```

Arguments

x	a vector
criterion	a function which takes a vector of length *n* and returns an integer between 1 and *n*-1

Value

tree	A list of J matrices, where J represents the number of "scales". Each matrix is of size 5 x (the number of UH coefficients at a given scale). Each column (= vector of length 5) contains an Unbalanced Haar coefficient in the following format: 1st component - an index of the coefficient; 2nd component - the value of the coefficient; 3rd component - time point where the corresponding UH vector starts; 4th component - last time point before the breakpoint of the UH vector; 5th component - end point of the UH vector.
smooth	the "smooth" component of x, equal to `sum(x) / sqrt(n)`, where n is the length of x

Author(s)

Piotr Fryzlewicz

See Also

inner.prod.max, inner.prod.max.p, best.unbal.haar.bu

Examples

```
best.unbal.haar(rnorm(100), inner.prod.max.p)
```

---

best.unbal.haar.bu *Best bottom-up Unbalanced Haar decomposition*

Description

The function finds the "best" bottom-up Unbalanced Haar (UH) decomposition of the input vector x.

Usage

```
best.unbal.haar.bu(x, stretch = length(x))
```

Arguments

x	a vector
stretch	at each iteration, only the first 1:stretch elements of the current input vector (whose length decreases by one with each iteration) get scanned in the search for the worst-fitting fine-scale Unbalanced Haar wavelet

Value

detail	A matrix of size 3 x n-1, where n is the length of x, containing the detail coefficients of x in the order they were chosen. Each column corresponds to a single coefficient and contains, from top to bottom: location of the coefficient, the associated weight, and the value of the coefficient.
smooth	the "smooth" component of x, equal to sum(x) / sqrt(n), where n is the length of x

Author(s)

Piotr Fryzlewicz

See Also

best.unbal.haar

Examples

```
best.unbal.haar.bu(rnorm(100))
```

---

hard.thresh *Hard thresholding of a top-down Unbalanced Haar decomposition*

Description

Presented with an object returned by best.unbal.haar, the function sets to zero those Unbalanced Haar coefficients which fall below a certain threshold sigma.

Usage

```
hard.thresh(buh, sigma = 1)
```

Arguments

buh	an object returned by best.unbal.haar containing the decomposition to be thresholded
sigma	the threshold (a positive scalar)

Value

a thresholded object, of the same class as buh

Author(s)

Piotr Fryzlewicz

See Also

best.unbal.haar, hard.thresh.bu

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar(x)
x.uh.th <- hard.thresh(x.uh)
x.uh.th.r <- reconstr(x.uh.th)
ts.plot(x.uh.th.r)
```

---

hard.thresh.bu *Hard thresholding of a bottom-up Unbalanced Haar decomposition*

Description

Presented with an object returned by best.unbal.haar.bu, the function sets to zero those Unbalanced Haar coefficients which fall below a certain threshold sigma.

Usage

```
hard.thresh.bu(buh.bu, sigma = 1)
```

Arguments

buh.bu	an object returned by best.unbal.haar.bu containing the decomposition to be thresholded
sigma	the threshold (a positive scalar)

Value

a thresholded object, of the same class as buh.bu

Author(s)

Piotr Fryzlewicz

See Also

best.unbal.haar.bu, hard.thresh

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar.bu(x)
x.uh.th <- hard.thresh.bu(x.uh)
x.uh.th.r <- reconstr.bu(x.uh.th)
ts.plot(x.uh.th.r)
```

`inner.prod.iter` *Inner products with Unbalanced Haar wavelets*

Description

For an input vector of length *n*, the function computes inner products between the input vector and all possible *n*-1 Unbalanced Haar vectors of length *n*.

Usage

```
inner.prod.iter(x)
```

Arguments

`x`	a vector of length *n*

Details

The computation is iterative and is performed in computational time *O(n)*.

Value

a vector of length *n*-1, containing inner products between `x` and consecutive Unbalanced Haar wavelets of length *n*

Author(s)

Piotr Fryzlewicz

Examples

```
inner.prod.iter(rnorm(100))
```

inner.prod.max *Unbalanced Haar wavelet which maximises the inner product*

Description

The function finds the Unbalanced Haar vector which yields the largest (in absolute value) inner product with the input vector.

Usage

```
inner.prod.max(x)
```

Arguments

x	a vector

Value

The index where `abs(inner.prod.iter(x))` is maximised. If two or more maxima are found, the `med` of their locations is returned.

Author(s)

Piotr Fryzlewicz

See Also

`inner.prod.iter`, `med`, `inner.prod.max.p`

Examples

```
inner.prod.max(c(rep(0, 100), rep(1, 200)))
```

inner.prod.max.p *Unbalanced Haar wavelet which maximises the inner product*

Description

The function finds the Unbalanced Haar vector which yields the largest (in absolute value) inner product with the input vector, amongst those Unbalanced Haar vectors whose breakpoint is located between 100(1-p)% and 100p% of their support.

Usage

```
inner.prod.max.p(x, p = 0.8)
```

Arguments

x	a vector
p	a scalar in (0.5, 1]

Value

The index where `abs(inner.prod.iter(x))` is maximised on the subinterval `(1+floor((1-p)*n)):ceiling(p*n)`, where `n` is the length of `x`. If two or more maxima are found, the `med` of their locations is returned.

Author(s)

Piotr Fryzlewicz

See Also

`inner.prod.iter`, `med`, `inner.prod.max`

Examples

```
inner.prod.max.p(c(rep(0, 100), rep(1, 200)), .55)
```

---

med *Median*

Description

The function computes the median of a vector. Unlike `median`, it is guaranteed to return a value which is a component of the input vector.

Usage

```
med(x)
```

Arguments

x	a vector

Value

a scalar defined as `quantile(x,.5,type=3)[[1]]`

Author(s)

Piotr Fryzlewicz

Examples

```
med(1:4)
median(1:4)
```

reconstr *Reconstruct a top-down Unbalanced Haar decomposition*

Description

Reconstructs a vector from its top-down Unbalanced Haar decomposition stored in an object returned by `best.unbal.haar` or `hard.thresh`.

Usage

```
reconstr(buh)
```

Arguments

buh	an object of the type returned by `best.unbal.haar` and `hard.thresh`

Value

the inverse Unbalanced Haar transform of buh

Author(s)

Piotr Fryzlewicz

See Also

`best.unbal.haar`, `hard.thresh`, `reconstr.bu`

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar(x)
x.uh.th <- hard.thresh(x.uh)
x.uh.th.r <- reconstr(x.uh.th)
ts.plot(x.uh.th.r)
```

reconstr.bu *Reconstruct a bottom-up Unbalanced Haar decomposition*

Description

Reconstructs a vector from its bottom-up Unbalanced Haar decomposition stored in an object returned by `best.unbal.haar.bu` or `hard.thresh.bu`.

Usage

```
reconstr.bu(buh.bu)
```

Arguments

buh.bu	an object of the type returned by best.unbal.haar.bu and hard.thresh.bu

Value

the inverse Unbalanced Haar transform of buh.bu

Author(s)

Piotr Fryzlewicz

See Also

best.unbal.haar.bu, hard.thresh.bu, reconstr

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar.bu(x)
x.uh.th <- hard.thresh.bu(x.uh)
x.uh.th.r <- reconstr.bu(x.uh.th)
ts.plot(x.uh.th.r)
```

---

uh *Denoising via top-down Unbalanced Haar*

Description

Given an input vector of the form "signal + iid Gaussian noise", the function estimates the noise level via Median Absolute Deviation, finds the best top-down Unbalanced Haar decomposition (according to the selection rule criterion), thresholds it with the universal threshold, and performs the inverse Unbalanced Haar transform to yield an estimate of the signal.

Usage

```
uh(x, criterion = inner.prod.max)
```

Arguments

x	a vector of the form "signal + iid Gaussian noise"
criterion	a function which takes a vector of length n and returns an integer between 1 and n-1

Value

an estimate of the signal

Author(s)

Piotr Fryzlewicz

References

P. Fryzlewicz (2007) "Unbalanced Haar technique for nonparametric function estimation". *Journal of the American Statistical Association*, 102, 1318-1327.

See Also

`uh.bu`, `best.unbal.haar`, `inner.prod.max`, `inner.prod.max.p`, `hard.thresh`, `reconstr`

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est <- uh(x)
```

---

uh.bu *Denoising via bottom-up Unbalanced Haar*

Description

Given an input vector of the form "signal + iid Gaussian noise", the function estimates the noise level via Median Absolute Deviation, finds the best bottom-up Unbalanced Haar decomposition, thresholds it with the universal threshold, and performs the inverse Unbalanced Haar transform to yield an estimate of the signal.

Usage

```
uh.bu(x, stretch = length(x))
```

Arguments

`x`	a vector of the form "signal + iid Gaussian noise"
`stretch`	at each iteration, only the first `1:stretch` elements of the current input vector (whose length decreases by one with each iteration) get scanned in the search for the worst-fitting fine-scale Unbalanced Haar wavelet

Value

an estimate of the signal

Author(s)

Piotr Fryzlewicz

References

P. Fryzlewicz (2007) "Unbalanced Haar technique for nonparametric function estimation". *Journal of the American Statistical Association*, 102, 1318-1327.

See Also

uh, best.unbal.haar.bu, hard.thresh.bu, reconstr.bu

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est <- uh.bu(x)
```

---

unbal.haar.vector *Unbalanced Haar vector*

Description

Computes the non-zero part of an Unbalanced Haar vector with a given start-, break- and end-point.

Usage

```
unbal.haar.vector(a)
```

Arguments

a	a three-component vector of integers such that a[1] <= a[2] < a[3]. The three components specify, respectively, the start point, the time point just before the breakpoint, and the endpoint of the desired Unbalanced Haar vector.

Value

the non-zero part of the corresponding Unbalanced Haar vector

Author(s)

Piotr Fryzlewicz

Examples

```
unbal.haar.vector(c(1, 1, 2))
unbal.haar.vector(c(2, 5, 12))
```

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